Mosmod Basics

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1 Introduction

Mosmod is based on the idea that when you mosaic a set of images it is a good idea to take into account all of the associations between the images to derive image to image models. The situation is like photogrammetric "bundle adjustment" and offers advantages over more common image to base map transformations that do not take into account overlapping areas or other matches between images.

In the formulation that follows we have used the terminology of "Page's" which are usually images or a map or the section of the world that is being mapped. In the Mosmod approach, there is one Page nominated as the "Base Page" or "Reference Page". The Base Page can be looked at as the collating base for the Mosaic or simply a base coordinate system underlying the modelling.

The total number of Pages will be written as N_{page}+1. That is, N_{page} Pages and one Base Page. Each page is assumed to have a geometric model mapping the data in the page to the base page of the form:

Geometric Model (G_j):

$$x_b = G_j(x_j, \mu_j)$$

where:

- x_j is the position of a pixel or spatial object in Page j
- μ_j is a set of parameters for the model in Page j and
- x_b is the mapped position in the Base Page corresponding to x_j

The assumption is that many of the pages overlap partially or completely and that equivalent geometric objects can be located in the overlaps. For any two Pages, identified objects are the "Tie Points" between the images. They can theoretically be patches as well as points but here it assumed the data are assigned to a point – such as the centroid.

Points in common between Page j and the Base Page are simply called Control Points.

Formally, if there are N_{jb} points common to Page j and the Base Page and there are N_{jm} points identified in the overlap between Page j and Page m then geometric modelling of the "Target Page" model (Page j) is seeking to adjust any parameters of G_j (μ_j) and the other Pages (μ_l, l=1,N_{page} $l \neq j$) such that:

$$\begin{aligned} G_j(\mu_j, x_{jk}) &\approx x_{bk} && \text{for } k = 1, N_{jb} \\ G_j(\mu_j, x_{jl}) &\approx G_m(\mu_m, x_{ml}) && \text{for } l = 1, N_{jm} \end{aligned}$$

The parameters are not, of course, only defined by the immediately specified relations such as may be found interactively or by a correlation method between selected image pairs. All and only pages that are "connected" with the page(s) of interest are ones that may be involved in the joint modelling[1].

The model also assumes that all "transitive" ties are computed and used in the constraints. By a "transitive" tie is meant one where ties have been identified between Pages j and m and also between those points in Pages m with others in Page k. This implies "transitive" ties between Pages j and k.

The desirable situation for mapping the Pages into the Base Page is therefore:

$$\begin{aligned} &G_j(\mu_j, x_{jk}) \approx x_{bk} && for\ \ j = 1, N_{page}, k = 1, N_{jb} \\ &G_j(\mu_j, x_{jl}) \approx G_m(\mu_m, x_{ml}) && \forall\ unique\ (x_{jl}, x_{ml}), l = 1, N_{Tie} \end{aligned}$$

where N_{tie} is the total number of different Tie points in the connected set of Pages which includes points in the Base Page that are identified with points in Pages other than j.

The parameters can be fitted by minimising functionals that measure goodness of fit, regularisation ("smoothness") and complexity. An example of how this works with a linear model is shown below which illustrates how the connections and mappings can be organised.

In this way, all of the models and overlap constraints will be involved simultaneously in the solution of the system. Only the choice of Base Page and/or Target Page will change between different cases

1.1 Examples

Assume that the image pages use geometric models defined by a polynomial, spline or other linear function. This would be the case with most satellite data or airborne data if the Pages were frames from which the major geometric distortions had been taken.

In this case, the Base Page can be the map and the assignments of points to the Base Page are "Ground Control Points" or GCPs. In this formulation, the issue is to fit the GCPs in each frame or Page as well as to match the overlapping Pages to the same degree of accuracy in the coordinate frame of the map.

If the goodness of fit criterion is least squares then the solution is given below. The advantage here is that, just like "Bundle Adjustment" the GCPs that you have between a Page and the map can pass control into adjacent Pages through the tie points. The success of this can be tested by analysing the consequent least-squares matrices as described below.

Conversely, the Base Page can be an individual image and the Map included among the Pages. Then a transformation is obtained from the Map to the image Page. Such an "inverse"

[1] By "connected" here is meant the transitive closure of the graph containing Page j that is formed by defining an undirected link between two pages when they overlap (and have Tie points) and then "close" the graph such that if A and B overlap and C overlaps B (but not A) then A and C overlap with "distance" 2.

mapping is important when it is necessary to find a pixel corresponding to a Map coordinate – such as in image resampling. With Mosmod, it is also the case that all of the Pages tie together with equal accuracy in the coordinates of the image Page.

This formulation also covers the case of time series data where the mosaic is a set of frames at different times that are closely registered. In many systems, each image is registered to a base page separately or to its nearest time image(s). However, by forming Tie points (by automatic correlation methods if possible) it is possible using the complete formulation to simultaneously register all the images pairwise to each other. This is a great advantage for time series products where ties between Pages are generally more crucial than accurate location in a map.

It is especially useful when some of the image Pages in the time series have features that can sometimes be located and not at others – such as fallow fields or when cloud or pointed images exclude features in specific Pages. Such missing matches can be well compensated by the additional ties.

1.2 "Forward" and "Inverse" Mappings

It is important to distinguish between "forward" and "inverse" mappings. A "forward" mapping is FROM a Page TO the Base Page. An "inverse" mapping is FROM the Base Page TO a Page. In Mosmod, the inverse map is obtained simply by choosing the given Page as a new Base Page and establishing the model that relates the previous Base Page to the given (new Base) Page.

The roles of base page, overlaps and reference Pages can be illustrated by considering the simple case of only three Pages. In the geometric case consider that there are three Pages A, B (eg two images) and M (eg a Map). The images and the map have pair wise selected control points, some of which are identified as GCPs (common to one of A and B and the Map) or Tie points (common to A and B).

The Mosmod approach resolves around choosing the "base Page". That is where all the geometric transformations map.

Case 1 Forward transformation of images to Map:

The base Page is the map M.

Transformations are:

FROM A TO M
FROM B TO M

Constraints are:

Fit to map GCPs in Page A are small;
Fit to map GCPs in Page B are small;
Differences between mapped Tie points in overlap between A and B are small in the Map coordinate system (eg metres).

In this case, the map GCPs form the "right hand sides" of the least squares equations. The result is that both the transformations (their coefficients) are available after the solution is obtained. So for either A or B you can go FROM pixel, line to Easting, Northing. The transformations from A and B to the Map will be “improved” by the constraint that they also agree in the overlap.

Case 2 Inverse Map to image A:

The base Page now is image A

Transformations are:

FROM B TO A
FROM M TO A

Constraints are:

Fit to image A GCPs in Page B are small;
Fit to image A GCPs in Page M are small;
Differences between mapped Tie points in overlap between B and M are small in the coordinate system of A (eg pixels).

Now it is really just the same. The ties between A and B are measured in the coordinate system of A as are those between B and M and M and A. The key is to see that the ties between M and B are being measured in the coordinate system of A.

Out of this you get the two transformations. Often you do not keep the transformation FROM B TO A but you could and I have done many applications where these cross-transformations are very useful.

Again, this mapping should be “improved” by the way it ensures that the image B and the Map are well registered at the same time as the transformations from A to B and A to M are established.

Case 3 Inverse map to image B:

The base Page now is image B.

Transformations are:

FROM M TO B
FROM A TO B

Constraints are:

Fit to image B GCPs in Page A are small;
Fit to image B GCPs in Page M are small;
Differences between mapped Tie points in overlap between A and M are small in the coordinate system of B (eg pixels).

Now it all follows as above. Note that the ties between A and M are in the coordinate system of B. It is all balanced in this way.

Again, you may only keep the transformation from M to B.

Obviously, A and B are acting as “reference” images for the transformations between the other image and the map. For example, if image B were a SPOT panchromatic image and Image A were a Daedalus (DATM) image then the transformation between the Map and the SPOT image could use control outside the DATM run to get high accuracy and there could be many Tie points generated between the SPOT image and the DATM image. In the end the transformations between the DATM image and the map may be all that is desired.

This approach can be used with both a highly constrained approach such as polynomial fitting or with a highly flexible approach like triangulation.

1.3 Realising the code

The basic situation for Mosmod involves creating a collection of Coordinate files – one for each Page. These files will contain selected points from the different “Pages” (i.e. images and/or a map). It is also essential to identify the matches between images as the “same” feature.

In the former microBRIAN system where Mosmod was used extensively, each “coordinate” or point associated with a feature has a unique number and identified points in different Pages are associated by having the same coordinate number. That is, a feature is uniquely identified irrespective of the Page in which it is initially located. If other systems are used some way to carry out this identification must be found.

Mosaicking involves setting up models FROM each Page TO another based on the coordinate files which preserves overlapping relationships with some or all of the overlapping Pages. The fitted models are then used to navigate the separate Pages for location of data or resampling. In the case of navigating an image, the model needed has the FROM Page as the image and the TO Page as the map. Then the transformation indicates the geometric coordinates of a pixel. In the case of resampling (eg with the microBRIAN REMAPPER) the transformation needed has the FROM Page as the map and the TO Page as the image of interest so that the pixel from which data are to be taken can be located for a specific map coordinate.

Suppose there are N+1 Pages. The first step in Mosmod is to select the Coordinate files and set up the Incidence Matrix *OverLap*:

$$Overlap = \left[n_{ij} \right]$$

where n_{ij} is the number of coordinates in common between Pages *i* and *j*. At the *same* time, the number of independent overlapping points ($\max_j$) with other images should be computed. This matrix is “closed” transitively and used to assess the connection “distance” between pairs of images.

The next step is to choose a FROM Page and a TO Page and the order of the model to be fitted. The model will be formed to map FROM the coordinate system of the FROM Page TO the coordinate system of the TO Page. If the FROM Page index is *f*, the TO Page index is *t* and the order of the model to be fitted is *p* then the following checks must be carried out:

1. Check whether $n_{tf} \geq p$. If it is then the other check can be skipped.

2. Otherwise, check whether $\sum_{j \neq f} n_{fj} \geq p$. If so, then the model may be feasible due to the extent of tie point control and can be attempted.
3. Otherwise, the model is not possible to fit as planned and should not be tried until something is done about it – basically by getting more control.

The *N* Pages other than the TO Page can be rearranged (sorted) so that the FROM Page is number 1 and the others are in increasing connection distance from the FROM Page. If the system will not fit into the possible space, Pages can then be deleted starting back from *N*. This needs re-computing and checking in some cases.

The Pages can also be separately checked for model feasibility. That is, $\sum_{k \neq j} n_{jk} \geq p$ for each *j*.

If a Page model is not feasible, the order of the model should be reduced until it is or the Page should be removed. At this point, fitting the set of models is feasible.

1.4 Setting up the Sytem

Let the vector α_{jt} represent the *p* parameters of the model for the j'th Page for $j = 1, N$ mapping the *j*'th Page to the TO Page. That is, the model for Page j is taken to have the form:

$$z_t(x) = \sum_{k=1}^{p} \alpha_{jtk} \, \phi_k(x)$$

where x is general notation for a point in the j'th Page to which the model is attached and z_t is the value of the mapped point in the TO Page. In our models, there is one such equation for each coordinate of the geometry.

The complete linear system can be written in matrix form as:

$$\begin{bmatrix} A_1 & 0 & . & . & 0 \\ 0 & A_2 & . & . & 0 \\ . & . & . & . & . \\ 0 & . & . & 0 & A_N \\ B_{12} & -C_{12} & 0 & . & 0 \\ B_{13} & 0 & -C_{13} & . & 0 \\ . & . & . & . & . \\ B_{1N} & 0 & . & 0 & -C_{1N} \\ 0 & B_{23} & -C_{23} & . & 0 \\ . & . & . & . & . \\ 0 & B_{2N} & . & 0 & -C_{2N} \\ . & . & . & . & . \\ . & . & . & . & -C_{N-2\,N} \\ 0 & . & . & B_{N-1\,N} & -C_{N-1\,N} \end{bmatrix} \begin{bmatrix} \alpha_{1t} \\ \alpha_{2t} \\ . \\ . \\ \alpha_{Nt} \end{bmatrix} \approx \begin{bmatrix} z_{1t} \\ z_{2t} \\ . \\ z_{Nt} \\ 0 \\ . \\ 0 \\ . \\ . \\ . \\ . \\ . \\ 0 \\ 0 \end{bmatrix}$$

Where, the matrices A, B and C are evaluations of the model basis functions at the relevant points. In this system of equations, the *N* sets:

$$A_j \, \alpha_{jt} \approx z_{jt}$$

are the systems leading to the solutions for each Page being separately mapped to the Base Page. In this case, the (*i*,*k*) element of the matrix A_j is $\varphi_k(x_i)$ for rows i=1,n_{jt}.

These are the equations that would be obtained by the program MODEL in microBRIAN. The MosMod development is to introduce the tie point interactions implied in the full system above.

The numbers of lines in the first *N* row blocks are $n_{1t}, n_{2t}, ..., n_{Nt}$. If any of these is zero, the block is taken as missing. This would occur if there were no points common between the image in question and the TO image (or Base Page). Similarly, there are $N(N-1)/2$ potential row blocks in the lower section of the system. The number of lines in each of these is n_{ij} where the index *i* runs from 1 to $N-1$ and the index *j* runs from $i+1$ to *N*. Again, if any pair have no overlapping points the block row is taken as missing.

The block matrices represent the mapping of the coordinates of the Tie points into the coordinate system of the Base Page (or TO Page). That is:

$$B_{jk}\alpha_{jt} - C_{jk}\alpha_{kt} \approx 0$$

This represents the equality of the points in the overlap between Pages j and k (k>j) in the coordinate system of the Base or TO Page as measured by small residuals.

1.5 Solving by Least Squares

The normal equations (neglecting weights for now) for the least squares solution are:

$$\begin{bmatrix} D_1 & -H_{12} & -H_{13} & . & -H_{1N} \\ -H_{12}^T & D_2 & -H_{23} & . & -H_{2N} \\ -H_{13}^T & -H_{23}^T & D_3 & . & . \\ . & . & . & . & -H_{N-1N} \\ -H_{1N}^T & -H_{2N}^T & . & -H_{N-1N}^T & D_N \end{bmatrix} \begin{bmatrix} \alpha_{1t} \\ \alpha_{2t} \\ . \\ . \\ \alpha_{Nt} \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ . \\ . \\ b_N \end{bmatrix}$$

where:

$$D_j = A_j^T A_j + \sum_{k=j+1}^{N} B_{jk}^T B_{jk} + \sum_{k=1}^{j-1} C_{kj}^T C_{kj}$$

$$H_{ij} = B_{ij}^T C_{ij} \quad (j > i)$$

and

$$b_j = A_j^T z_{jt}$$

This system is easily solved and statistics generated using Block Cholesky. This is done in the microBRIAN program Mosmod. In the case of nonlinear models this solution will be applied to the Jacobian of the model in a Marquardt iteration. Again, Block Cholesky is the best means of obtaining a solution.

Note that if the individual Pages are matched to the Base Page the separate normal equations would be:

$$A_j^T A_j \alpha_{jt} = A_j^T z_{jt}$$
$$or$$
$$D_j \alpha_{jt} = b_j$$

The tie points generally add to the strength of the diagonal of the matrix and can overcome a lack of control in individual Pages by the strong tying between Pages.