Review of Geometric Distortion Compensation in Fish-Eye Cameras

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Abstract — **The majority of computer vision applications assume the pin-hole camera model. However, most optics will introduce some undesirable effects, rendering the assumption of the pin-hole camera model invalid. This is particularly evident in cameras with wide fields-of-view. The aim of distortion correction is, therefore, to transform the distorted view of wide-angle cameras to the pin-hole perspective view. Fish-eye cameras are those with "super-wide" fields-of-view, e.g. those cameras with fields-of-view of up to 180 degrees. However, these lenses exhibit severe forms of distortion. The most evident of these is radial distortion, but several other distortions, such as uneven illumination and inaccurate estimation of the centre of distortion, should also be considered when using a fish-eye camera. In this paper, we review and discuss methods of correcting radial and other distortions for fish-eye cameras and illustrate the effect of these methods on a test image exhibiting multiple types of distortion.**

I Introduction

The ideal model for an imaging device is the pin-hole camera model. However, cameras rarely follow the pin-hole model, due to undesirable effects caused by lens elements. Fish-eye cameras deviate particularly strongly from the pin-hole model, introducing high levels of geometric nonlinear distortion. Thus, camera calibration and distortion correction are important pre-processing tasks for computer vision applications. Not only does it make images captured by the camera more visually appealing to the human observer, it is also necessary for any computer vision tasks that require the extraction of geometric information from a given scene.

This paper reviews and discusses several methods for correcting the various distortions introduced by fish-eye lenses. Section II of this paper describes radial distortion, including various models proposed for this distortion. Section III describes the use of the distortion models to correct for radial distortion. Section IV describes the other geometric distortion considerations of using fish-eye cameras.

II Radial Distortion

Radial lens distortion causes image points on the image plane in the fish-eye camera to be displaced in a nonlinear fashion from their ideal position in the pin-hole camera model, along a radial axis from the *centre of distortion* (COD) in the image plane. The visual effect of this displacement in fish-eye optics is that the image will have a higher spatial resolution in the foveal area, with the spatial resolution decreasing nonlinearly towards the peripheral areas. In figure 1, this results in the text towards the centre of the image being sampled with more spatial points (pixels) than the text towards the periphery of the image.

For normal and narrow field-of-view cameras, radial distortion can be considered negligible for most applications. However, in wide-angle and fish-eye lenses, radial distortion can cause severe problems, not only visually but for further processing in applications such as object detection, recognition and classification. Additionally, the radial distortion introduced by fish-eye lenses does not preserve the rectilinearity of objects in its transformation from real-world coordinates to the image plane.

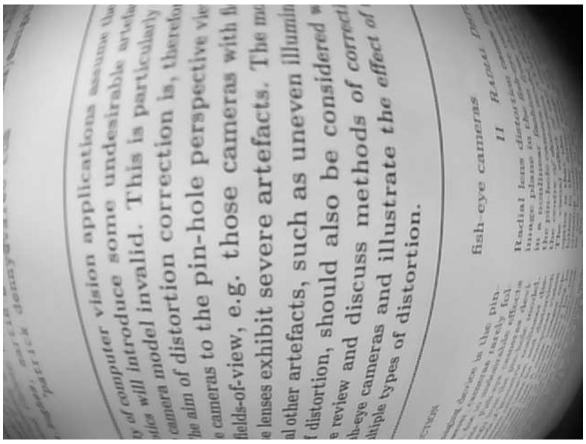


Fig. 1: Photographic example of fish-eye distortion. Text towards the centre of the image is sampled with more pixels than the text towards the periphery.

a) Polynomial Models of Radial Distortion

The use of polynomials to model radial distortion in lenses is standard practice [4, 5, 6, 7]. From an embedded implementation point of view, polynomials are desirable as they do not require the estimation of functions based on *log* and *tan* operations (although, with the use of look-up tables, this advantage is lessened). Practical problems with polynomial models arise due to the fact that there is no analytical method to invert them, i.e. there is no general method to invert a forward model to its inverse for use in correction of radial distortion.

1) Odd Polynomial Model: The standard model for radial distortion is an odd polynomial, as described by Slama in [4] and subsequently used in, for example, [5, 6, 7]:

$$\begin{aligned} r_d &= r_u + \sum_{n=1}^{\infty} \kappa_n r_u^{2n+1} \\ &= r_u + \kappa_1 r_u^3 + \ldots + \kappa_n r_u^{2n+1} + \ldots \end{aligned} \tag{1}$$

where r_d is the distorted radius, r_u is the undistorted radius and κ are the polynomial coefficients. While there is no general analytical method of inverting (1), an inverse to the fifth order version can be approximated as described by Mallon and Whelan [6]:

$$r_u = r_d - r_d \left(\frac{\kappa_1 r_d^2 + \kappa_2 r_d^4 + \kappa_1^2 r_d^4 + \kappa_2^2 r_d^8 + 2\kappa_1\kappa_2 r_d^6}{1 + 4\kappa_1 r_d^2 + 6\kappa_2 r_d^4} \right) \tag{2}$$

2) The Division Model: Fitzgibbon introduced the *division model* in [8]:

$$\begin{aligned} r_u &= \frac{r_d}{1 + \sum_{n=1}^{\infty} \kappa_n r_d^{2n}} \\ &= \frac{r_d}{1 + \kappa_1 r_d^2 + \ldots + \kappa_n r_d^{2n} + \ldots} \end{aligned} \tag{3}$$

It should be noted that this is inherently an inverted model, i.e. it models the undistorted radial distance of a point as a function of the distorted radial distance of that point. While (3) is similar in form to (1), note that it is not an approximation to an inversion of the standard polynomial model. Rather, both are approximations to the camera's true distortion curve. A first order version of this model is often used when circle fitting is employed to calibrate a lens, as this allows distortion estimation to be reformulated as a circle-fitting problem for which many algorithms are available [2, 3].

3) The Polynomial Fish-Eye Transform: (1) and (3) can be used to describe distortion in standard, non-fisheye lenses. However, it is generally considered that these polynomial models are insufficient to describe the level of distortion introduced by fish-eye lenses. Shah and Aggarwal have shown in [9] that even when using a seventh order version of (1) to model fish-eye radial distortion, considerable distortion remains, to the extent that that they had to use a model with greater degrees of freedom. Therefore, a polynomial that uses both odd and even coefficients (instead of simply one or the other) can be used to model the radial distortion introduced by a fisheye lens [11, 12]. :

$$\begin{aligned} r_d &= \sum_{n=1}^{\infty} \kappa_n r_u^n \\ &= \kappa_1 r_u^1 + \ldots + \kappa_n r_u^n + \ldots \end{aligned} \tag{4}$$

Basu and Licardie introduced a very similar model, *Polynomial Fish-Eye Transform* (PFET), in [10]. However, they allowed a 0th order term, i.e.:

$$r_d = \kappa_0 + \kappa_1 r_u^1 + \ldots + \kappa_n r_u^n + \ldots \tag{5}$$

The benefit of using the coefficients beyond the fifth order is generally considered negligible in this instance [10].

b) Non-Polynomial Models of Fish-Eye Radial Distortion

In this section we introduce several fish-eye distortion models that are not based on a polynomial approximation of the fish-eye lens distortion curve. One of the more notable advantages of using non-polynomial models over the polynomial models is that they are, in general, more readily inverted using analytical methods for application in distortion correction.

1) The Fish-Eye Transform: Basu and Licardie proposed the *Fish-Eye Transform* (FET) in [10]. This model is based on the observation that a fish-eye image has higher resolution in the foveal areas and lower resolution towards the peripheral areas:

$$r_d = s \ln (1 + \lambda r_u) \tag{6}$$

where s is a simple scalar and λ controls the amount of distortion across the image. The inverse of this model is given by:

$$r_u = \frac{e^{\frac{r_d}{s}} - 1}{\lambda} \tag{7}$$

2) The Field-of-View Model: Devernay and Faugeras described the *Field-of-View* (FOV) model, based on a simple optical model of a fish-eye lens, in [13]:

$$r_d = \frac{1}{\omega} \arctan\left(2 r_u \tan\frac{\omega}{2}\right) \tag{8}$$

where ω is the angular field-of-view of the ideal fish-eye camera. They point out that ω may not correspond to the *actual* camera field-of-view, since the fish-eye optics may not exactly follow this model. Additionally, they point out that this model may not always be sufficient to model the complex distortion of fish-eye lenses. In these cases, (1) can be used with $\kappa_1 = 0$ before applying (8).

The inverse of this model is:

$$r_u = \frac{\tan(r_d \omega)}{2\tan(\frac{\omega}{2})} \tag{9}$$

3) Perspective model: Another frequently used model for radial distortion is the *perspective model* described in [14]:

$$r_d = f \arctan\left(\frac{r_u}{f}\right) \tag{10}$$

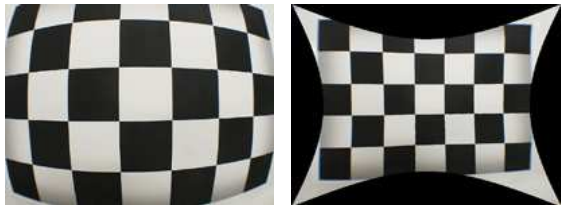
where f is the *apparent* focal length of the fish-eye camera. The inverse of this model is:

$$r_u = f \tan\left(\frac{r_d}{f}\right) \tag{11}$$

The *apparent* focal length f does not necessarily equate with the *actual* focal length of the fish-eye camera, since the fisheye optics often include several different groups of lenses that affect the actual physical focal length of the fish-eye camera.

III Radial Distortion Correction

Radial distortion correction is the process by which points in the distorted fish-eye image are transformed to points in the undistorted image. It is possible that radial distortion can be optically corrected using appropriate combinations of lenses. However, according to Bogner in [15], the amount of distortion that can be corrected by lenses is physically limited by the refractive, reflective, and transmissive characteristics of the materials from which they are made. The best wide angle optics produce acceptable rectilinear images at fields-of-view up to about 110 degrees. Therefore, for fish-eye lenses with fields-of-view greater than 110 degrees, it is necessary to perform some form of post-processing to convert the image to the rectilinear model. Figure 2 shows an example of the correction of radial distortion. This section outlines some key considerations when carrying out fish-eye compensation.



(a) Distorted checkerboard (b) Corrected checkerboard

Fig. 2: Example of fish-eye correction. Note that in (a) the lines in the image are arcs of circles, whereas in (b) they are "straightened".

a) Calibration

In general, the correction of radial distortion involves a calibration procedure to determine the parameters of one of the fish-eye models, described in the previous sections, to fit the distortion of a particular fish-eye lens camera. Then, the distortion can be corrected by inverting the model and transforming each pixel in the image according to that model (the exception is the division model described by 3, which is an inverse model as it stands). This results in a second, "undistorted" image.

The majority of calibration procedures make use of a calibration diagram with known geometry in 3-d space. Features from the calibration diagram are used to calibrate the camera, such as corners, dots, lines and circles or any other feature that is easily extracted from the image. This is known as *photogrammetric* calibration, and there are many methods using calibration diagrams [5, 9, 10, 16, 17, 18]. Alternatively, a *self-calibration method* can be employed, whereby the calibration system has no *a priori* knowledge of the scene. Rather, the method extracts the necessary information from an arbitrary scene, via point correspondences in multiple view geometry, circle-fitting or other suitable methods [1, 8, 19, 20].

b) Vacant Pixels

Due to the essential "stretching" effect of distortion correction (*undistortion*), resultant images will contain many vacant pixels that will not have been mapped during the undistortion procedure, as shown in figure 3. Interpolation methods can be used to overcome this, as implemented in [14, 16, 21].

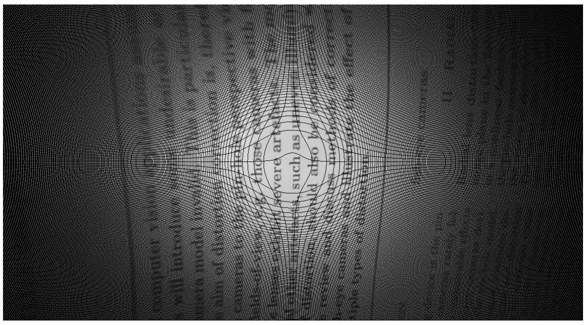


Fig. 3: Vacant pixels in this undistorted image are visible as black lines.

As an alternative, *back-mapping* may be used [12]. Instead of mapping every pixel in distorted space to undistorted space, back-mapping does the inverse. Back-mapping calculates the location of the pixel in undistorted space and uses a forward transform model to determine that pixel's location in distorted space. This overcomes the problem of vacant pixels because every pixel in the undistorted image will be assigned a value, as shown in figure 4.

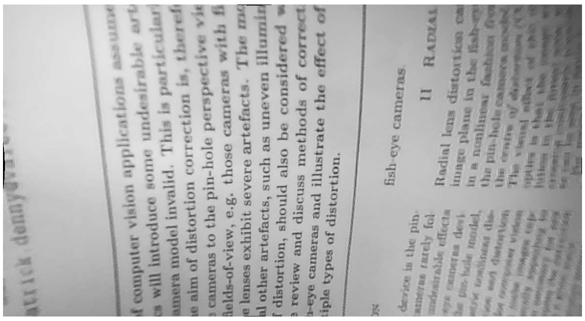


Fig. 4: Figure 1 undistorted, using back-mapping to overcome the problem of vacant pixels.

IV Other Fish-Eye Lens Effects

a) Centre of distortion

Radial lens distortion is the displacement of image points along a radial axis from a single point on the image plane. This point is known as the *centre of distortion* (COD), and this does not necessarily align with the image sensor centre. Therefore, in order to be able to fully model and correct radial lens distortion, it is necessary to accurately determine the COD. Physically, the COD is the point at which the optical axis of the camera lens system intersects the image plane. Inaccurate estimation of the COD will introduce additional radial distortion, as well as a degree of tangential distortion (described in section IV:b).

The estimation of the COD is only relevant in wide-angle camera systems that display reasonable degrees of radial lens distortion. In fact, according to Ruiz *et al.* [22], the location of the COD in cameras with small to moderate fields-of-view is irrelevant. Methods to estimate the COD are described in [7, 10, 11, 12].

b) Tangential Distortion

Tangential distortion is a design imperfection usually due to low quality optics and internal camera misalignment. According to Mallon and Whelan [6], this causes a geometric shift of the image along, and tangential to, the radial direction through the principal point.

There are two primary cause of tangential distortion: inaccurate COD estimation, as described in the previous section, and *thin prism distortion*. According to Weng *et al.* [23], thin prism distortion arises from imperfection in lens design and manufacturing as well as camera assembly, and causes a degree of both radial and tangential distortion. [4] and [9] both give mathematical models to deal with tangential distortion. Stein demonstrated in [24] that low levels of tangential distortion can be compensated for just by using COD estimation. Several other researchers make the assumption that other causes of tangential distortion can be considered negligible [3, 5, 6, 13, 25].

c) Uneven Illumination

In cameras with considerable fields-of-view, such as fish-eye and wide-angle lenses, there is a non-linear loss of illuminance towards the periphery of an image due to the structure of the camera lens system. This is noticeable in figure 1, where the extremities of the image are considerably darker than the central area. There are several causes of this distortion and several mathematical models that have been proposed to correct for uneven illumination [26, 27, 28].

A non-model based approach was proposed by Leong *et al.* in [29]. They observe that uneven illumination is an additive low frequency signal in the image, and use a Gaussian kernel low-pass filter to extract the uneven illumination pattern from a given image.

However, an alternative effective yet simple method of correcting for uneven illumination introduced by the camera is described in [30]. A uniformly illuminated white surface is imaged to find an intensity profile. The maximum intensity response is found, and a correction factor is determined for each pixel location using the following:

$$P_{lut}(i,j) = \frac{P_{ref,\max}}{P_{ref}(i,j)} \tag{12}$$

where $P_{lut}(i,j)$ is the correction factor for a given pixel location to be stored in a LUT, $P_{ref}(i,j)$ is the intensity response for the same pixel location and $P_{ref,\max}$ is the maximum value of $P_{ref}(i,j)$. Any image taken with the same camera can be corrected by simply multiplying each pixel value by the corresponding LUT value:

$$im_{corr}(i,j) = im_{orig}(i,j)\, P_{lut}(i,j) \tag{13}$$

Figure 5 shows an example of an image corrected using this method.

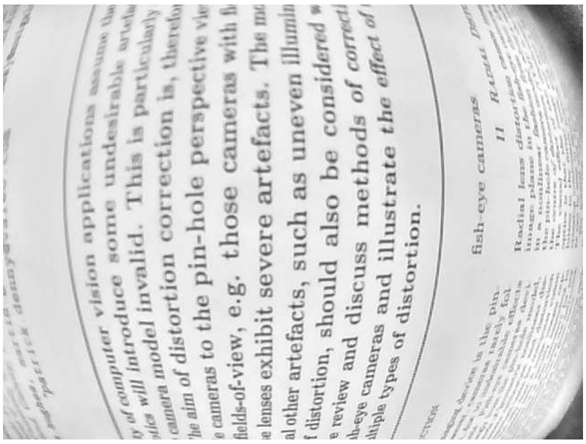


Fig. 5: Figure 1 with the uneven illumination removed.

V Summary

In this paper we have described several of the issues involved with using fish-eye cameras, and discussed several methods to correct for these effects. If all the necessary distortions are removed from a fish-eye image, the result is an image that, for many applications, accurately approximates the desired rectilinear model.

Radial distortion is by far the most evident geometric distortion introduced by fish-eye lenses, and is the effect that is most associated with fish-eye cameras. However, we have also described how other unwanted effects need to be considered, such as COD, tangential distortion and uneven illumination.

Figure 6 shows an example of the correction of all distortions described in this paper . Comparison of this figure with the original distorted image in figure 1 illustrates the combined effectiveness of the methods described.

The review presented in this paper is part of an overall project goal of providing means of improving fish-eye image quality. Future work in this area, for the authors, will be in providing a subjective method of comparing the potential accuracy of given geometric distortion correction and calibration algorithms.

VI Acknowledgements

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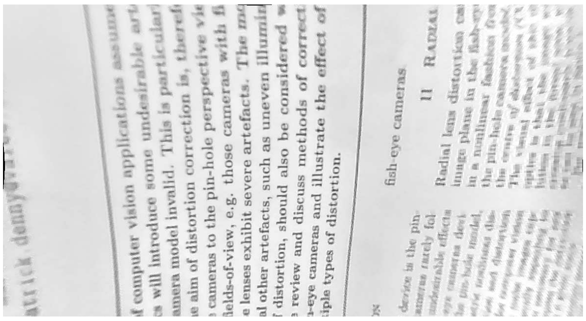

Fig. 6: Figure 1 with all distortions removed. Radial distortion is removed using (10) and back-mapping, the center of distortion is calculated as pixel location $(245.3, 305.1)$ and uneven illumination is removed using (12) and (13)

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