DETERMINING PROJECTION AND SCALE FOR MAPS IN EARLY ATLASES OF CHINA HELD BY THE NATIONAL LIBRARY OF AUSTRALIA

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Abstract: The National Library of Australia holds some maps and atlases from the 17th and 18th Century when cartography was between the period of manual construction of projections and the rise of mathematically defined projections. This paper shows how, despite their age, digital scanning and mathematical projections can often be used to geocode the maps and put them into a modern image processing format. This can create opportunities to study the comparative principal map scale, accuracy and historical contents of the collections. The digital maps can also be rescaled, reprojected and merged in mosaics using image processing. Where the atlases have gazetteers of points these can also be brought into the same framework and used to assess the underlying or potential accuracy of the map construction. The valuable nature of the collections introduces limitations and how these may be taken into account to allow the widest range of cases to be processed is discussed.

INTRODUCTION

This paper discusses the establishment of map projection and principal scales in mid-17th and early 18th century maps of China, taken primarily from well-preserved early editions of two significant atlases, available at the National Library of Australia (NLA). It then outlines briefly how the parameters found may be used to help analyse the maps in a modern setting using image processing and Geographical Information System (GIS) tools. The atlases in question were published in Europe using information and materials provided by Jesuit missionaries in China. The atlases and the information that accompanied them dramatically increased the cartographic and general geographic knowledge of China and East Asia in Europe, and are an important source for historical study of China in the late Ming and early Qing periods. They also represented a major outcome for the China mission of the Society of Jesus. (Spence 1980; Brockey 2007).

The first major export of detailed Jesuit geography and maps to Europe occurred between 1625 and 1665. During this time the Jesuit Brother Martino Martini (1614-61) returned from China and put together a set of province maps that were drafted and printed in 1655 by the well-known European cartographer Johannes Bleau. Martini's maps and associated descriptions of China's provincial geography were collected in his book *Novus Atlas Sinensis* (Martini 1655). His information also provided important inputs to the compendium by Athanasius Kircher (SJ, 1602-80). Kircher never travelled to China but wrote extensively of it in *China Illustrata* (Kircher 1667). The NLA has some fine holdings of Martini's atlas and **Fig. 1.** shows one of the maps of the collection, *Imperii Sinarum nova description*, which includes the 15 Ming provinces as well as Japan, Korea and Southeast Asian countries as known at the time. The projection seems to be an equidistant Conic with central meridian through Beijing and it includes global longitudes apparently measured in degrees east of the Azores.

The second major transfer of detailed cartographic information from China's interior occurred in the early 18th century. A group of French Jesuit mathematicians were contracted by the Kangxi Emperor between 1704 and 1719 to undertake a major ground survey of China and its adjacent territories. The

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survey is described in detail by Jean-Baptiste du Halde (SJ, 1674-1743) in a massive four volume work (du Halde 1735). Du Halde's book was later translated into English and printed by Edward Cave (1741). The cartographic material was developed by Jean Baptiste Bourguignon d'Anville from the material sent back by the Jesuits under contract to du Halde.

As described by Mario Cams (2014), the maps were printed from engraved plates and included with volume 4 of du Halde's book. However, due to the inconvenient format and high cost, the attached maps were not the popular source in Europe at the time, which was rather a "pirate" printing under d'Anville's name (but without his permission) in The Hague in 1737, the *Nouvel atlas de la Chine, de la Tartarie Chinoise et du Thibet*. Although d'Anville believed the quality of the Hague printing was lower than his original maps, it was not until 1785 that he brought out a high quality and better formatted set of all of the maps (using the original plates) under the same title. The NLA has a copy of this 1785 atlas which has provided material to study the second, Kangxi, survey in this paper.

The 1785 d'Anville atlas represents a less direct relationship between map maker and publisher than the Martini atlas. The Jesuits sent material to du Halde in Europe on a number of occasions between 1719 and 1723 (Cams 2014). The original surveys had been collated into detailed maps, made into a mosaic and re-drawn to form the basis of a copper-plate version at larger scale than the base province collations. The details of this activity is described by Yee (1994), and the relationships between the materials created in China and those published in Europe are provided by Cams (2014). From the latter, it seems likely that the maps of d'Anville's atlas were all – or, at least, mostly – based on the copper-plate mosaic re-drawn at various scales. The base maps and the copperplate map all had place names in Chinese whereas the maps published by d'Anville had transliterated names for a subset of the places as provided by the Jesuits. Nevertheless, every transformation in the process seems to have been made with great care and attention to detail.

Fig. 2. shows a scanned map sheet of the general map of China produced by d'Anville. It includes more recent information from Central Asia and information from the Russian-backed first Bering expedition (1725-28). Otherwise the information was obtained during the Jesuit-led Kangxi survey of 1704-19. Korea and Tibet as well as some other areas west of the 15 provinces were not visited by the Jesuits, and the map was based on information provided by Chinese surveys. Like the Martini map (**Fig. 1.**), this map is apparently on an equidistant Conic projection with central meridian through Beijing. There is also a scale bar relating Chinese long and short *li* to French long and short *lieue* (leagues).

The five maps (two of China as a whole and three of Shaanxi province) used in this paper are a selection intended to both illustrate the methods and also represent the opportunities that exist:

1. Martini map of China (**Fig. 1.**) printed as a map sheet by Blaeu and identical that published in Martini (1655). Scanned by NLA;
2. d'Anville map of China (**Fig. 2.**) from the 1737 'pirate' edition. Scanned by NLA;
3. 1655 Martini map of Xensi [Shaanxi] province (**Fig. 5.**). Photographed by NLA.
4. 1785 d'Anville map of Chen-Si [Shaanxi] province (**Fig. 6.**). Photographed by NLA;
5. 1721 Kangxi Jesuit map of Shensi [Shaanxi] province (**Fig. 7.**) from woodblock printed maps; scanned by the Library of Congress from Regis *et al*. (1941).

The d'Anville maps were based on the surveys collated into the Kangxi maps of which the 1721 maps are likely the most accurate and detailed. The primary resource for the d'Anville maps was a copperplate mosaic of all of the maps that was engraved in China by Mario Ripa SJ in 1719. The copperplate map was on a Sinusoidal projection as demonstrated by Wang (1991), and most of the d'Anville maps are re-projections and/or re-drawings from this base map.

Figure 1. Martino Martini [1648-55] *Imperii Sinarum nova descriptio*, W.J. & J. Blaeu, Amsterdam. (National Library of Australia MAP RM 214, online at https://nla.gov.au/nla.obj-231209112)

Figure 2. J.B.B. d'Anville (1737), *Carte la plus generale et qui comprend la Chine, la Tartarie chinoise*, *et le Thibet*. (National Library of Australia MAP RM 3521, online at https://nla.gov.au/nla.obj-232293356)

The main purpose of the present paper is to investigate the specific projections and scales involved in the general and provincial maps and demonstrate how that knowledge may be used to help analyse them for specific historical studies and purposes. The methods are the same as those used in Jupp (2017) to estimate parameters for the 1721 woodblock printed edition of the Jesuit Kangxi maps (Regis *et al.* 1941). By establishing the parameters of the projections, the maps are able to be re-transformed into other projections and displayed as presentations in software such as Google Earth. Displaying them in this setting with their published gazetteers of places allows the geography of the time to be studied in detail as well as the relative scales and accuracies to be established.

ESTABLISHING MAP PROJECTION AND SCALE FOR OLD MAPS

Geocoding old maps

The processing discussed here adds geocoding to old maps to exploit the many ways that geocoded digital spatial data can be analysed and combined with other data. The first stage of the process is to obtain a digitised map. Many old maps are now available through library websites as scanned or photographed images from printed map sheets or printing plates. The ones best used here are those that are as "metric" and distortion free as possible. That is, the maps are as far as possible not rotated or distorted by optics, paper stretch and folds or page curvature from binding in atlases. Of course, with very old maps a major limiting factor in minimising these distortions is the need to protect and preserve the valuable materials. Map curators are justifiably resistant to scanning rare and valuable materials. This preliminary step involves some care, as described clearly in Daniil *et al.* (2003) and Tsioukas *et al.* (2012). It is assumed for the present paper that the inputs are as good as can be provided given state of technology and the balance of the different factors.

The scanned or photographed map is stored as an image, perhaps as a tiff file or similar image format. An image is a rectangular array of pixels (picture elements) assumed to be the same size and referenced by column and row numbers or (using terminology more common in image processing) pixel and line coordinates. Using 'pixel' can sometimes be a little confusing as every element of the array is technically a pixel and not just those in lines. The terminology comes from imaging such as digitizing where a line of pixels (the raster) is scanned/recorded and then the next line is recorded etc., so that 'pixel' refers to a particular pixel in a recorded line. Image display since early radar and the first TV has usually been organised in this way. In addition, an image pixel often has a number of data values associated with it. These are sometimes called 'bands' so that a grayscale image has one band and a colour image has three. The ability of computers and display technology to process and analyse images rapidly and with a high degree of sophistication has led to growth of the science of image processing.

Maps are a representation of the surface of the globe on a flat surface and therefore involve some form of projection. Since early times (Snyder 1993) and at least since the time of Ptolemy (*ca.* AD 150, see Stevenson 1932 quoted by Snyder 1993) map makers have used simple projection surfaces to map the world. Projecting onto a plane, a cylinder or cone and (in the case of the cylinder and cone) unwrapping the surfaces to planar maps has been the base for map making. Until the mid-18th century this was done by manual drafting, whereas since that time the world of map projections has become mathematical, computational and a part of the science of cartography. In modern times the process has become digital. The projection based methodology described here is not specific to old maps, although this paper more attention is paid to steps and approaches demanded by old maps and are less significant for modern maps. The methods have certainly been used with profit with 19th and early 20th century maps as well as the more ancient maps.

In all cases the first step is to identify the possible (class of) projection that the image may have. The use of projection planes based on cylinders, cones and planes has led to projections being described and classified variously as Conic, Cylindrical, Azimuthal and in various categories in between. The projection class has usually been identified by the way that the meridians and parallels of the globe appear in the projection. Pearson (1990, Tables 1 & 2 in ch.10) lists the various features that characterise a projection type. For example, radiating straight line meridians and curved parallels indicate a Conic type, while parallel straight line parallels with straight or curving meridians are likely of a cylindrical or pseudo-cylindrical type. As illustrated by Wang (1991), an experienced person can usually identify the class of projection of a map by these observations.

For any one of the possible projections, there is a set of mathematical equations relating position in the projection plane (e.g. in metres) with the latitudes and longitudes of the parallels and meridians. The parameters of the projection are then decided by 'fitting' the mathematical projection to the data. The suitable data are, in the first place, where the lines of parallels and meridians cross in the scanned map. However, most maps involved in our work were accompanied by a gazetteer of tabulated places with latitudes and longitudes. If these places are plotted onto the image of the map using the first estimate of the projection it is possible to investigate the 'plotting accuracy' of the points. Plotting accuracy includes not only the accuracy of manual plotting by the map maker, but also the effects of paper distortions such as stretch and folding, as well as digitising and printing issues. These latter issues are also present for modern maps. The points that seem reasonably well plotted can then be combined with the grid-crossing data and used to refine the projection. Other information, such as common boundary points between maps in different map sheets can also be thrown into the mix at this stage. The result is a relationship between the position in the image and the position in the projection plane as well as parameters of the projection that converts the map locations into latitudes and longitudes of the parallels and meridians. If various projections are possible, the best fitting one is selected as the most likely.

This approach has behind it a model of how maps were likely to have been drawn in earlier times. First, the map maker would have constructed and drawn the lines of constant latitude and longitude at a defined spacing and on a sheet of large enough size to accommodate the area of interest and the map detail desired. For the maps discussed here, the manual constructions involved preceded the mathematical development of map projections. After that, the gazetteer points were most likely plotted using the grid of meridians and parallels locally to calculate the locations. Finally map details would be entered – possibly scaled from other maps or field notes in relation to the plotted gazetteer places or field calculated latitudes and longitudes. When map scale and projection are changed, the most likely method would be for the contents of the cells defined by the grids to be redrawn manually. The best maps would then re-plot the gazetteer points as reference, but often information would simply scale across locally. When adjacent areas occurred on different sheets it is likely the map makers also took reference coordinates for features in common to enable the different maps to be later merged into a mosaic.

Once the maps are geo-coded, many operations originally carried out manually can be done by computer. One such step is to determine the scale of the map. The map maker would probably have defined this when constructing the original grid but due to many things including manual issues, scale adjustments to fit the area of interest and later map distortions, the final effective scale is often different. Map 'scale' is usually defined as: "The relationship between a distance on a map, chart, or photograph, and the corresponding distance on the Earth". However, it is best to separate this composite of scales into two main factors called principal scale and local scale (Pearson 1990). These two factors multiplied together become the composite 'scale' as defined above. Principal scale is the ratio of distance in the map to distance in the projection plane. It describes how the original projection of a part of the earth's surface onto the projection plane is shrunk to fit onto a table or onto the page of an atlas. The amount of shrinking involved is the principal scale. A map has a single principal scale and usually a scale bar based on it.

An equivalent way to look at this is to consider that the globe is shrunk down so that the projected area of interest fits onto the desired sheet. For example, if the principal scale is to be 1:10,000,000 this means that if the projected surface area is obtained from a globe with diameter 1.274m it would coincide with the map sheet. The second scale factor (local scale) is the ratio of distance in the projection plane to distance in the globe (or equivalently, distances in the map in relation to the shrunken globe). It concerns the distortions associated with a projection. Most maps include lines where they are at 'true scale'. These are true local scale but local scale will always vary from place to place away from the true scale lines in any projection. In this paper the estimation of scale is always principal scale. It is done as described later by making measurements on the maps or from accurate estimates of the actual scanned image DPI. These are related to the pixel size in the projection plane as estimated by the fitting procedure described above.

MATHEMATICAL METHODS USED TO ESTABLISH PROJECTION AND MAP SCALE

Projections used

The mathematical expression of a projection is a transformation from map coordinates for the meridians (the longitude λ) and parallels (the latitude φ) to metric coordinates x and y of the form:

$$\begin{aligned} x &= f(\lambda,\varphi) \\ y &= g(\lambda,\varphi) \end{aligned} \tag{1}$$

That is, (λ,φ) are the coordinates on the sphere and (x, y) the coordinates on the projection plane.

The simple, or equidistant, Conic Projection

Using Snyder (1993) as reference, the equations for the simple Conic with a single standard parallel φ_1 and standard longitude λ_0 are:

$$\begin{aligned} x &= \rho \sin\theta \\ y &= \rho_0 - \rho\cos\theta \\ \rho &= sR(\cot\varphi_1 + \varphi_1 - \varphi) \\ \rho_0 &= sR\cot\varphi_1 \\ \theta &= \sin\varphi_1(\lambda - \lambda_0) \end{aligned} \tag{2}$$

R is the radius of the (spherical) earth. For the work described here it is the radius of the Authallic sphere corresponding to the WGS84 spheroid. The standard parallel φ_1 is where the cone is tangent to the surface of the globe and is a line of 'true local scale'. The term s is the principal scale factor. If s = 1 then the x and y values are in metres on the projection surface relative to the earth globe. The determination of the 'principal scale' of the map is to find the value of s corresponding to x and y being in (e.g.) cm on the printed map. Alternatively, it is the amount by which the globe must be shrunk for the projected surface to match the size desired for the map.

In appearance (c.f. **Figs. 1 & 2.**), a simple Conic has concentric circles to represent the parallels and radial straight lines (from the apex or pole) to represent the meridians. Among the auxiliary terms in Eq. 2, ρ is the radius of the circle for latitude φ, ρ_0 is the radius for latitude of φ_1, the standard parallel, and θ defines the angle between the central radial meridian and the radial meridian at longitude λ. Constructing these figures geometrically was the task faced by the early map makers.

The radial meridian lines are all true local scale but the circular lines of constant latitude are only true local scale at the standard meridian . In other cases, the lengths of arcs on the circle are longer than the corresponding arcs on the globe with the difference increasing away from the standard parallel.

A simple Conic with two standard parallels (where the cone cuts the surface at two lines of true local scale) can also be used to fit the maps as an alternative to the Conic with one standard parallel. The overall local scale distortion on the circular arcs can be reduced by using two standard parallels, and it was commonly done in regional maps. Although the method is not examined in detail here, the case of two standard parallels can be inferred from fitting a model with one parallel. Where it occurs it will be discussed and the details left for another document.

The Sinusoidal Projection

The mathematical definition in this case (Pearson 1990; Snyder 1993) is:

$$\begin{aligned} x &= s\,R(\lambda-\lambda_0)\cos\varphi \\ y &= s\,R\varphi \end{aligned} \tag{3}$$

The Sinusoidal is an equal area projection (Pearson 1990). It is also called 'pseudo-cylindrical' as its parallels are equally spaced and parallel. At the equator, the *x*-axis has length extremes of $\pm\pi sR$ or total length equal to the (scaled) circumference of the earth. The *y*-axis has extremes $\pm\pi sR/2$ or a total length of half the scaled circumference (**Fig. 3.**). The Sinusoidal has true local scale along every parallel and on the standard meridian with longitude λ_0 .

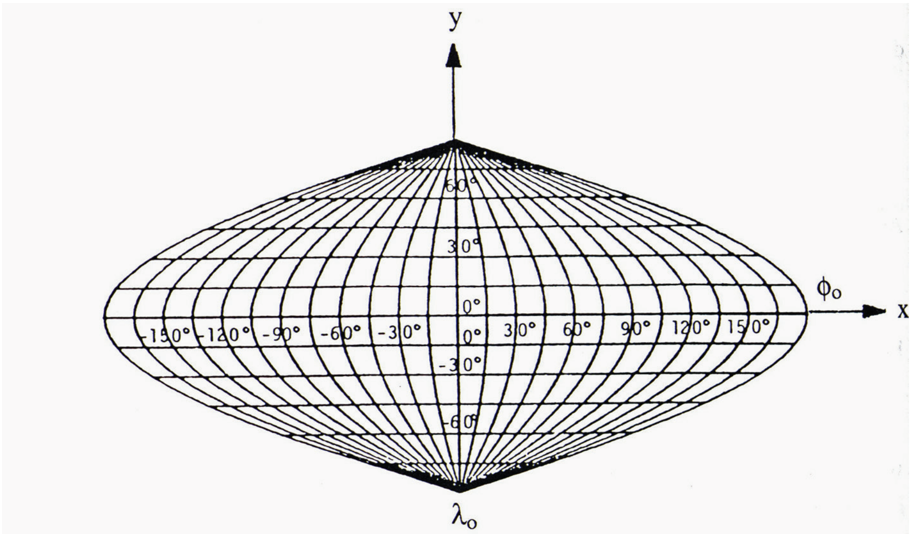

Figure 3. Sinusoidal Projection of the whole Earth sphere (from Pearson 1990)

The Sinusoidal can be used as a whole globe map and is the projection in which NASA delivers its earth observation products from the MODIS mission (MODIS Land, n.d.). The Sinusoidal projection was certainly used in the original Kangxi maps (Wang 1991) as well as d'Anville's province maps of China.

The Trapezoidal Projection

The Trapezoidal projection is much older and was established well before the Sinusoidal and, for the time we are discussing, was only still used in mid-latitude regions. It is included here as it was the projection some originally felt the Kangxi maps had used and is a candidate for the projection Martini used for his province maps. Wang (1991) showed unequivocally by measurements that the correct projection for the copper-plate edition of the Kangxi maps was the Sinusoidal, but with the Martini maps it is less clear.

In the Trapezoidal projection, all parallels and meridians are ruled straight lines. The Trapezoidal projection for the whole sphere has horizontal, equally spaced, parallels and a central meridian about which the meridians are symmetric. It therefore has a roughly similar appearance to the Sinusoidal shown in **Fig. 3.** except that all meridians are straight lines, need not meet at the poles, and the local scale on the equator may not be true. The definition is that the lengths along parallels are true local scale at two standard parallels symmetric between northern and southern hemisphere. The result can then be used as a global projection and some early world maps were of this form. By 'true' is meant that the total length of the parallel at reference latitude φ is $2\pi sR\cos\varphi$. This is the length of the same parallel on a sphere with radius *R*. Lengths of sections on the chosen parallels therefore have the same length on the globe and the same length on the same segments in the Sinusoidal projection. In the Sinusoidal projection the lengths of all parallels and lengths of segments on parallels are true to local scale – by definition.

The most common use of the Trapezoidal projection has been in mid-latitudes to map only the area between the two standard parallels at which the local scale on the earth is true. The local distortions were also minimized (as is also the case for the Sinusoidal) if the standard meridian (λ_0) is in the middle of the area of interest. To define the local projection mathematically, if the top (φ_T) and bottom (φ_B) parallels are given and the longitude and latitude (λ,φ) are between the parallels then the *y*-value is the same as for the Sinusoidal:

$$y = s\,R\varphi \tag{4}$$

The *x*-values of the meridian crossing with longitude λ at the reference latitudes can be written:

$$\begin{aligned} x_T &= sR(\lambda-\lambda_0)\cos\varphi_T \\ x_B &= sR(\lambda-\lambda_0)\cos\varphi_B \end{aligned} \tag{5}$$

It follows that the *x*-value for a ruled line between these crossings through (λ,φ) is:

$$x = x_T\frac{\varphi-\varphi_B}{\varphi_T-\varphi_B} + x_B\frac{\varphi_T-\varphi}{\varphi_T-\varphi_B} \tag{6}$$

Written out fully:

$$\begin{aligned} x &= sR(\lambda-\lambda_0)\left(\cos\varphi_T\frac{\varphi-\varphi_B}{\varphi_T-\varphi_B} + \cos\varphi_B\frac{\varphi_T-\varphi}{\varphi_T-\varphi_B}\right) \\ y &= sR\varphi \end{aligned} \tag{7}$$

It is clear from this expression that the degree to which the Trapezoidal and Sinusoidal are different at φ' in the interval (φ_B,φ_T) is the degree to which the linear interpolation of $\cos\varphi$ at φ' is different from $\cos\varphi'$. Simple calculations show that if $\varphi_T-\varphi_B$ is less than about 4-5 degrees, then the differences are small if not negligible – especially if the longitude of the location is not far from the central meridian.

To be specific, suppose the parallel mid-way between (φ_B,φ_T) is considered and set $\delta\lambda = \lambda-\lambda_0$ then the difference between where the λ meridian is in the Trapezoidal and where it would be in a Sinusoidal projection will be:

$$\delta x = sR\delta\lambda\left(\cos\left(\frac{\varphi_T+\varphi_B}{2}\right) - \frac{\cos\varphi_T+\cos\varphi_B}{2}\right) \tag{8}$$

This formula will be used later when discussing the Martini map of Shaanxi province.

For the extreme case where the reference parallels are the equator and the pole, the equations for the projection can be globally defined as:

$$x = s\,R(\lambda - \lambda_0)(1 - 2|\varphi| / \pi)$$
$$y = s\,R\varphi \tag{9}$$

In this case, the meridians meet at the poles and the equator is true local scale. But in between the meridians diverge significantly from true distance to the central meridian. Equation 7 above also shows how for arbitrary pairs of parallels the projected lines outside of the interval will not meet at the pole and will not be true local scale at the equator. However, a piecewise Trapezoidal global projection that is very close to Sinusoidal could be constructed by using a set of parallels of spacing of about 1 to 5 degrees of latitude (depending on map scale) and covering the Earth with separate ruled meridians in each interval. It is possible that the early manual constructions of the Sinusoidal were achieved in this way.

Wang (1991) used manual measurements of the copperplate edition of the Kangxi Map using 5-degree spacing and established its projection as Sinusoidal. More recently, Jupp (2017) analysed a woodblock printed version of the 1721 base maps from which the copperplate seems to have been generalised. The maps used 0.5-degree spacing of the graticule and it was found that the Sinusoidal fitted closely. It seems that by 1719 the technology of drawing sine curves must therefore have become very good.

Estimating Projection Parameters and Map Scale

The projection models can be used to establish the parameters of modern projections that fit the map, test one projection against other possible projections, and establish principal scale. This is done by determining the relationship between image coordinates and coordinates in the projection plane. If the map is not rotated, the scanned image should be congruent with the projection plane. Hence, if the image pixel number (i.e. column number) is denoted by j and the line number (i.e. row number) by i then the projection coordinates of the pixel (x, y) in units such as metres or centimetres can be expressed using four parameters as:

$$x = x_0 + h_x (j - 1)$$
$$y = y_0 + h_y (i - 1) \tag{10}$$

The parameters x_0 and y_0 are the projection coordinates of the top left corner of the image and h_x and h_y represent the column and row distance between image pixels. Later it will be found that h_y is negative. This is simply because line numbers increase in the $-y$ direction for most standard projections. Unless there is information to the contrary, the linear sizes of the pixel in the two dimensions are assumed to be the same as the absolute values of the spacing between pixels along lines and between lines. That is, there are no gaps between pixels and pixels do not overlap. In theory, h_x and h_y should be equal in magnitude if the scanning is metric and there is no paper distortion. However, they are normally left to vary freely to investigate possible local scale differences and errors due to (e.g.) paper distortions and folds. However, if they are significantly different beyond such causes, there is some issue with the projection which must be resolved.

The four parameters (x_0, y_0, h_x, h_y) can be estimated by finding points where the geographic coordinates (λ_k, φ_k) are known and the pixel and line values (j_k, i_k) are also known for match points indexed as k for $k = 1, N$. Common choices are the crossings of parallels and meridians in the map and places in the map gazetteer. The geographic coordinates of the points are converted to projection plane coordinates (x_k, y_k) using the projection equation.

$$x_k = f(\lambda_k, \varphi_k)$$
$$y_k = g(\lambda_k, \varphi_k) \tag{11}$$

These are matched with the pixel and line values as:

$$\begin{aligned} x'_k &= x_0 + h_x(j_k - 1) \\ y'_k &= y_0 + h_y(i_k - 1) \end{aligned} \tag{12}$$

Provided there are a sufficient number and spacing of match points, the values of the four parameters can then be found in the same units as the projection coordinates using minimum least squares. That is, by minimising the Total error defined as:

$$\begin{aligned} Xerr &= \left(\sum_{k=1}^{N} (x_k - x'_k)^2 / N \right)^{1/2} \\ Yerr &= \left(\sum_{k=1}^{N} (y_k - y'_k)^2 / N \right)^{1/2} \\ Total &= \left((Xerr^2 + Yerr^2) / 2 \right)^{1/2} \end{aligned} \tag{13}$$

Projection parameters such as the standard parallel φ_1 in the equation for the equidistant Conic can also be varied (in which case one uses nonlinear least squares) to establish the best fit value for any unknown parameters of the projection model.

To determine the principal map scale, some information about the physical sizes or lengths of features on the map page are needed. The two terms h_x and h_y are the sizes of the steps of one pixel in the *x* direction and one pixel in the *y* direction on the projection plane. These can be directly estimated (for example) if measurements of horizontal and vertical lines (including possibly the neat-lines and scale bar) are made. Then the number of pixels along horizontal lines or number of image lines crossed by a vertical line in the image are measured. These provide measured overall average sizes of a pixel p_x and p_y in map units (such as cm) in the *x* direction and in the *y* direction on the map. Writing h for the (geometric) mean of h_x and h_y in metres, and $psize$ for the equivalent (geometric) average size of a 'pixel' on the page in centimetres, the principal scale can be estimated as:

$$\begin{aligned} h &= \sqrt{h_x h_y} \\ psize &= \sqrt{p_x p_y} \\ Scale &= 1:100 \times h / psize \end{aligned} \tag{14}$$

The use of the geometric mean is to effectively match the areas of the pixels.

Alternatively, the principal scale can be obtained from the metric properties of the scanner or camera when they are known. A typical scanner uses a line of detectors to scan a line of pixels on a page. The distance between the view points of the detectors on the page defines the distance between pixels in the map. The scan line is moved forward by steps that define the distances between lines. If the scanner is well calibrated the two distances will be the same. The reciprocal of this distance is the scanner DPI. This measure of density can also often be established for a modern camera. In this case the image is captured by a CCD array of detectors and depending on focal length, field of view and camera optics a nominal DPI can be estimated. For the maps scanned and photographed here a nominal instrument DPI was provided. For example, if the scan is at 300 DPI with one pixel being one 'dot', then h_x and h_y in the projection would be equivalent to p_x and p_y values of 0.008467 cm (1/300 inch) on the digitized map. The equation for average scale given above can therefore be used with:

$$psize = \frac{2.54}{DPI} cm \tag{15}$$

If h_x and h_y are not equal then perhaps either the principal map scale or the DPI is not the same in the *x* and *y* directions. If the DPI is different in the two directions, the scanner settings or camera optics may not be well adjusted. Using measurements of the neat-lines or other vertical and horizontal lines can therefore help determine which of these situations occurs.

RESULTS FOR SELECTED MAPS

Various maps from the collections outlined in the introduction have been selected and analysed as described in the previous section. In all of the maps selected, longitudes are provided based on a standard meridian at Beijing (i.e. as degrees west or east of the Beijing meridian). This is the case for the general and province maps as well as for the gazetteer of places (Martini 1655; du Halde 1735). Longitudes east of the Azores are also shown in the general Martini map but in this and other exercises involving the two Jesuit map collections, the global longitudes were retrieved by giving the Beijing meridian a value of 116.391° E of Greenwich. This choice excludes inaccuracy and variation in the determinations of the Beijing longitude during these years. The maps were all developed and surveyed relative to the Beijing meridian as reference, and so it was felt biases due to changing astronomical observations of Beijing should be removed for accuracy assessments.

The two general maps exist as separate sheets and were scanned at 600 dpi using a flatbed scanner. They had earlier been photographed at 300 dpi and some comparisons are made between these cases for accuracy of determining principal scale and projection parameters. The provincial maps in atlases were photographed at 300 dpi. This was because the atlases could not be disbound. However, the careful techniques used seem to have minimised the distortions due to page curvature. The results compare favourably with the printed map sheets of the Kangxi map collection of 1721 (Jupp 2017) which werestored folded and scanned much later.

The 1655 general Martini map of China

The first map is Martini's collated map of the 15 Ming provinces of China (**Fig. 1.**). The map exists in the collection as a separate sheet and therefore has relatively few distortions and can be scanned using a flatbed scanner. Using the scanned image, the latitude and longitude crossing points were sampled to record line and pixel values in the relevant image and the simple Conic projection was fitted by least squares with the standard parallel being adjusted by nonlinear least squares. The results for the Martini map are summarised in **Table 1**.

Table 1. Martini map of China: result of fitting a simple Conic projection

Parameter	Value	Units
x_0	-2,657,105.74	m
y_0	2,025,271.05	m
h_x	367.75	m
h_y	-355.26	m
φ_1	31.88	degrees
X_{err}	2.018	km
Y_{err}	5.479	km
Total	3.748	km
Av pixel	361.50	m
Map scale = 1:	8,540,712	
Ratio h_x / h_y	1.0352	
DPI	600.09	
Map RMS	0.04389	cm

In **Table 1** (and for similar tables in the following sections with variations noted when they occur) the parameters and references to their calculation are defined as follows:

(x_0, y_0) are the coordinates in metres for the map projection at the top left-hand corner of the scanned image and the values (h_x, h_y) are the steps in *x* and *y* in metres ($-h_y$ indicates direction as well as magnitude). (Eq 10)
φ_1 is the latitude of the standard parallel in degrees. (Eq 2)
X_{err} and Y_{err} are the RMS errors separately in *x* and *y* in km and *Total* is total error (RMS of combined set of *x* and *y* errors) in km. (Eq 13)
Av pixel (average pixel) is the (geometric) average of the absolute values of (h_x, h_y). (Eq 14)
Map scale is the principal map scale calculated here using measured neat-lines. (Eq 14)
Ratio h_x / h_y is the ratio of absolute values of h_x to h_y.
DPI is the dpi of the scanner or camera and in this case was estimated by using the principal scale and inverting the equations provided earlier. (Eqs 14, 15)
Map RMS is the *Total* RMS error (above) scaled to centimetres on the map using the estimated principal scale.

To provide some visual indication of the way the observations are distributed and of the fit by the model, the plotted positions of the estimated (*x* , *y*) values and fitted values for the Martini map are very close as seen in **Fig. 4**.

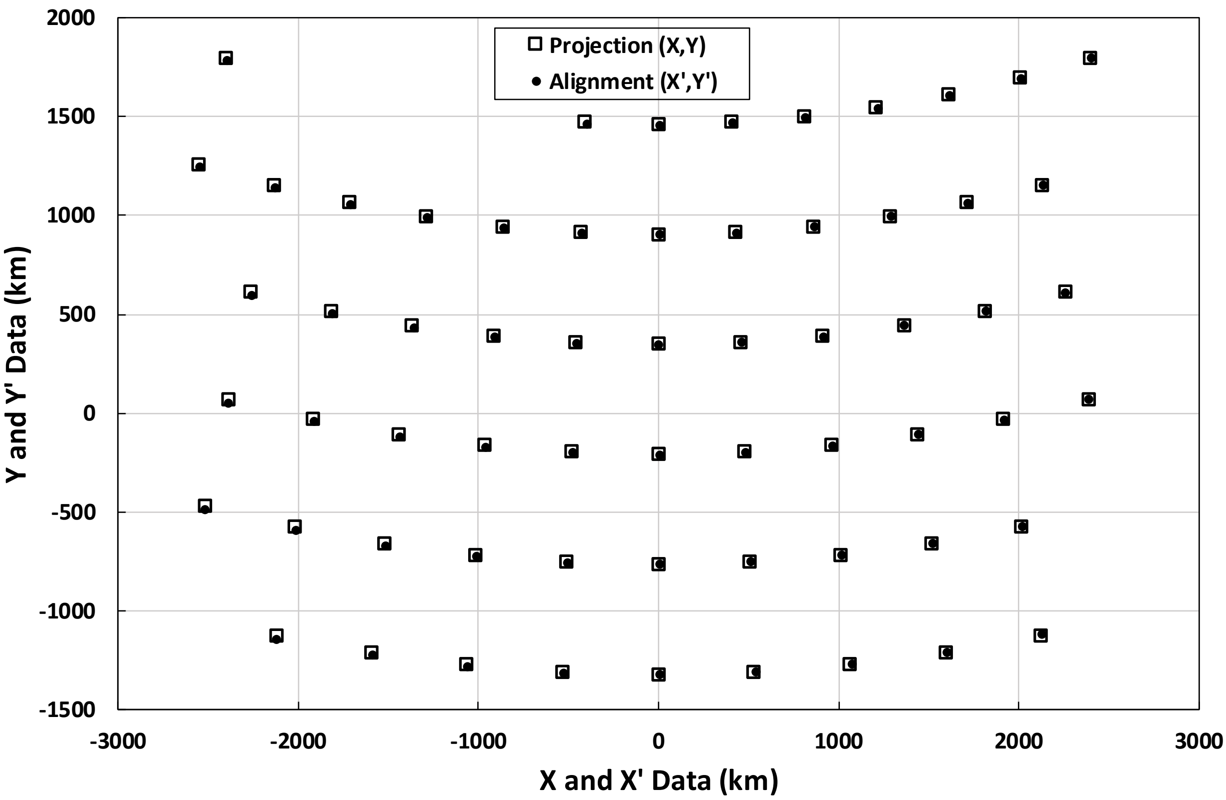

Figure 4. Plot of projection (X,Y) and alignment (X',Y') positions of the grid crossings in Martini's 1655 map of China *Imperii Sinarum nova description*, using the fitted equidistant conic model. The origin is at the standard parallel on the central meridian, units are km.

Based on neat-line measurements the estimated principal map scale was 1:8,540,000 to three significant figures. It seems likely that the particular choice of scale simply came about as a result of manually matching the target page size and the area to be covered by the graticule. The true scale parallel (φ_1 in Eq 2) is estimated to be 31.88ºN. This is in between the two middle (circular) parallels on the map.

Using the principal scale, the RMS variation at the crossing points in cm on the maps can be estimated at 0.043 cm. This is a very high plotting accuracy for manual plotting without mathematical equations and shows why the map maker was so highly regarded. For that reason it is very hard to distinguish differences between the exact and modelled Conics in **Fig. 4**. The close fit also suggests the map was in very good condition with little distortion. The Ratio h_x / h_y is discussed as it can indicate differences in map scales or differences in dpi values for the scanner or camera in *x* and *y*. The Ratio h_x / h_y value of 1.0352 is a significant departure from 1.0 and it will be discussed further in the next section together with the d'Anville map.

The map used to calculate the table was scanned at a nominal density of 600 dpi based on the scanner specifications, and was also measured to provide a value based on neat-lines. The estimated dpi value based on neat-line measurements (*DPI* = 600.09) and the nominal instrument setting (600 dpi) proved to be very close. The same Martini map had previously been digitized nominally at 300 dpi using photography and a similar table based on the 300 dpi digital map had given essentially the same results for the map parameters above. However, using the same measurements of the map neat-lines resulted in a more divergent estimated *DPI* of 295.7. This variation is not unexpected as there may be issues of range and focus that can alter estimated *DPI* of a single photograph relative to nominal dpi.

The 1737 d'Anville general map of China

The d'Anville map (**Fig. 2.**) also shows properties like those expected of a Conic type of projection with circular parallels and radial meridians. Like the Martini map, the longitudes are given as degrees to the east or west of the Beijing meridian. Again, to remove bias due to varying (and inaccurate) estimates of the Beijing longitude, a value of 116.391ºE of Greenwich was taken to convert the local differences to Beijing into global longitudes.

The map was a separate sheet and able to be scanned flat at 600 dpi. Table 2 shows the fitted parameters of simple conic projection, using the same definitions as defined previously for the Martini map.

Table 2. d'Anville map of China: result of fitting a simple Conic projeciton

Parameter	Value	Units
x_0	-5,004,799.48	m
y_0	2,955,825.90	m
h_x	448.05	m
h_y	-431.63	m
φ_1	33.79	degrees
X_{err}	7.942	km

Parameter	Value	Units
Y_{err}	4.779	km
Total	6.361	km
Av pixel	439.84	m
Map scale = 1:	10,381,949	
Ratio h_x / h_y	1.0381	
DPI	599.54	
Map RMS	0.06127	cm

The value for the reference latitude (φ_1 =33.79º) of the simple Conic projection in **Table 2** is significantly different from that of the Martini map (31.88º) with interchange of the values resulting in errors of 100–200 km. The standard error has not been calculated precisely but may be of the order of 0.1–0.2º. The estimated reference latitude is not too far from what seems to be the middle (circular) parallel at 35º.

The principal map scale of 1:10,380,000 suggests that the map maker was possibly aiming at 1:10,000,000 and the *RMS* accuracy of 0.061 cm on the map page is a very good plotting accuracy for a map of this size and the range of circular arcs. An estimated effective *DPI* of 599.54 based on the neat-line measurements again shows that the nominal and estimated dpi values for the scanned image are very close for the flatbed scanner. Originally, this map was also photographed at nominal density of 300 dpi and in that case, the measured neat-lines indicated the estimated *DPI* for the photographed image was 309.8, being again more variable relative to nominal dpi than for the scanned image as was found with the Martini map.

The ratio h_x / h_y is almost identical to that of the Martini map at 1.0381. One possibility for a departure from 1.0 would be that the scanner dpi is different in the *x* and *y* directions. That is, the values of the p_x and p_x of Eq 14 could possibly be different from each other. However, separating the neat-line measurements into vertical and horizontal cases showed no significant difference in pixel size ($p_x / p_y = 1.002$) and so the difference in this case is not due to the scanner. This finding was confirmed using the 300 dpi photography which resulted in essentially the same ratio of h_x / h_y as the scanner case but showed no differences in measured pixel size. Similar arguments can be made for the Martini map.

The explanation is that the two maps are, in fact, Conic projections with two standard parallels. The move to two standard parallels seems to have been made to improve the local scale of each map. The standard parallels for the d'Anville were 17.35º and 48.35º and for the Martini map they were 16.15º and 46.05º. As the one standard parallel model with the modified pixel sizes behaves identically in relation to geocoding as the two standard parallel model with square pixel values, a detailed discussion of the Conic projections and their local scales is left for another document.

The plot of the map crossings as digitised and estimated (not included) showed as close an approximation to the mathematical projection by the manually plotted projection as was seen in **Fig. 4**. Such an accuracy can be expected of the map makers involved in both cases and for a projection as straight forward as the simple Conic. The quality of the fit would also have been maintained by the maps being preserved as plates for the later reprinting rather than manually copied.

The 1655 Martini map of Xensi [Shaanxi] province

The copy of Martini's map of Xensi (i.e. Shaanxi, the predecessor of the three current provinces of Shaanxi, Gansu and Ningxia) used here is an atlas plate which has been photographed by the NLA at 300 dpi (**Fig. 5.**). The map does not have an internal graticule but only indicates the latitude and longitude on the margins. It seems from the images in the NLA catalogue that there is a separate standard meridian located centrally for each provincial map. The projected parallels of latitude are equally spaced and parallel in the map and the projected meridians show symmetric convergence across each image and around the central meridian. Such properties can be found with both Sinusoidal and Trapezoidal projections.

The information around the sides of the map are sufficient to fit a Sinusoidal but with few degrees of freedom (number of independent pieces of information minus number of parameters to estimate) left to compensate for error. Referring back to the equations for the Sinusoidal (Eq 3 and 10), the parameters are obtained as follows:

On the two sides are markers where the parallels are noted. Using any reasonable image processing software system it is easy to find the image row values at each marker on the two sides. The *y* values are known using the equation for the Sinusoidal given previously with the latitude values. Hence the equation:

$$y = y_0 + h_y(i-1) \tag{16}$$

can be fitted and used to estimate y_0 and h_y for this set data by using only values on the left and right sides of the map and least squares. As the borders at the top and bottom are not drawn at a standard latitude of the grid, this equation must be used to estimate the average interpolated latitudes of the top (φ_T) and bottom (φ_B) border lines.

Figure 5. Martino Martini (1655), *Xensi* [Shannxi] from *Novus Atlas Sinensis*.
(National Library of Australia MAP RA 300 plate 4, online at https://nla.gov.au/nla.obj-230903176)

The top and bottom border lines also have tick marks for the longitudes. The values are in degrees west of Beijing. Initially, the central meridian for the province is not precisely known. Given an estimate for the central meridian, the projected x values at top (x_{jT}) and bottom (x_{jB}) (where j indicates a tick marker) of the image at these longitudes can be estimated as:

$$\begin{aligned} x_{jT} &= R(\lambda_{jT} - \lambda_0)\cos\varphi_T \\ x_{jB} &= R(\lambda_{jB} - \lambda_0)\cos\varphi_B \end{aligned} \tag{17}$$

And the following equation is fitted:

$$x = x_0 + h_x(j-1) \tag{18}$$

The fit is made over the set of points at top and bottom of the map. A good estimate for the central longitude is usually easy to make and after a preliminary fit, λ_0 can be also made a variable in the fit for greater accuracy. However, to estimate λ_0 as well, the sums of squares of errors for both of the above cases are combined so that all of the estimates can vary and adjust.

The result of the fit of a sinusoidal projection is given in **Table 3**.

Table 3. Martini map of Xensi [Shaanxi]: result of fitting a Sinusoidal projection

Parameter	Value	Units
x_0	-626,119.44	m
y_0	4,581,593.96	m
h_x	231.86	m
h_y	-230.09	m
λ_0	105.24	degrees
X_{err}	1.357	km

Parameter	Value	Units
Y_{err}	0.411	km
Total	1.003	km
Av pixel	230.98	m
Map scale = 1:	2,728,067	
Ratio h_x / h_y	1.008	
DPI	300.00	
Map RMS	0.037	cm

The parameters are much the same as before except that the projection parameter λ_0 (Eq 3), the longitude of the central meridian, replaces φ_1 (the standard parallel of the Conic projection) as a parameter to estimate. Also, the nominal dpi of 300 is used to estimate the principal map scale as no map measurements were taken. Unlike the general maps based on Conic projections, the ratio h_x / h_y indicates the scales in *x* and *y* are very close to equal. The latitude of the central meridian is close to 105º. It is possible that it was chosen to be 105º by the map maker. The *RMS* (0.037 cm) indicates an accurate plotting of the grid, and the accuracy is not unlike the accuracy achieved for the Conic projections of the general maps.

The principal map scale is about 1:2,730,000 which is near what may be expected given the details being plotted and when compared with other maps. Based on Jupp (2017), the Kangxi base province maps were all the same at near 1:1,940,000 principal scale, and according to Wang (1991), the principal map scale of the collated copperplate mosaic was 1:1,400,000. When all of the Martini provinces are analysed it will be clear whether the principal map scale is similar for all of his maps and what overall drawing precision is being obtained. Visually it appears that the principal map scale and central longitude can both vary from province to province.

A thorough investigation of whether the maps were intended to be Sinusoidal or Trapezoidal will at least need the model to be applied to many more (preferably all) of the provinces. However, a preliminary study using Xensi shows that the Trapezoidal model with the estimated latitudes at top and bottom of the map (φ_T and φ_B) as true local scale parallels is not distinguishable from the proposed Sinusoidal. This is due in part to the map parameters only being provided around the edge but also due to the relatively small latitude difference between φ_T and φ_B and to a central meridian being in the middle of each map. The close fit of the Sinusoidal model also shows that the maps are close to true local scale at φ_T and φ_B. Since, when using the Trapezoidal model, the parallels will not be true local scale between the parallels, this shows that if the correct model is Trapezoidal then the standard parallels *must* be close to φ_T and φ_B and *not* at other parallels. Additional information can be obtained by using gazetteer places after they are first sieved for badly plotted points. They can likely help as whatever model was used must have guided their placement. This is certainly worth doing, if only to improve whichever model is chosen, but for Xensi it is unlikely to resolve the fundamental question.

The main reason for likely continued uncertainty is that, as shown in Section 2 (Eq 8), the point at which the ruled (Trapezoidal) meridian departs most from the Sinusoidal meridian in longitude is half way between the standard parallels at an edge as far from the central meridian as possible. In the Xensi map, the distance between the parallels is about 6 degrees and an edge is about 5 degrees from the central meridian. If the maximum difference is calculated using the formula previously derived (Eq 8), the maximum difference is found to be about 0.064 cm on the map page. This can be compared with the *RMS* error of about 0.037 cm as being similar to the average shift error across the page. That is, given the accuracies of the drawn map, it would be a marginal decision whether the projection is Sinusoidal or Trapezoidal even if the internal graticule were drawn. Based on this argument, the Sinusoidal model will most likely be used for any further investigations due to its consistency, scalability and flexible capacity to re-project and rescale the province data.

The same analysis as the above can, and should, be carried out for each of the province maps in the Martini atlas. By using varying central meridians and principal map scales, Martini reduced distortions but sacrificed being able to directly mosaic the province maps without re-drawing and re-projection. However, using the Sinusoidal models it is possible to easily re-project the provinces and at the same time make them consistent in principal scale. Additional constraints can also be used to help the mosaic. One is to use the Martini gazetteer places (after sieving out badly plotted points) to help fit the models. Another is to add boundary constraints to help improve the fit at province borders if they are combined into a mosaic. Such additional constraints have recently been successfully used for a mosaic of the 15 provinces using the 1721 Kangxi maps which can be accessed at the Kangxi Maps web page (n.d.).

The 1785 d'Anville Map of Chen-Si [Shaanxi] province

As with the Martini atlas, in addition to the general maps, the d'Anville atlas also has individual maps for each of the 15 provinces and for areas to the north and west of China. As noted by Cams (2014), these maps were most likely re-drawn from the copperplate map. For comparison with the Martini atlas, **Fig. 6.** shows the d'Anville map of Chen-si [i.e. Shaanxi] province.

The copper plate edition was engraved by Matteo Ripa in China in 1719 from a mosaic of earlier provincial base maps. It differed from the older collating base of woodblock printed maps (Regis *et al*. 1941) in a number of ways. It was at an enlarged principal scale; the graticule was at 1^{o} intervals for both parallels and meridians, rather than every half-degree; the mountains were more finely drawn with less clutter, mainly to show the catchments of rivers; the rivers were smoothly drawn with artistic balance; and the smaller drawn characters were excellent calligraphy. These aspects mostly carry over to the d'Anville maps. However, the Jesuit Brothers only provided d'Anville with a subset of transliterated place names. Close inspection of the d'Anville province maps shows there are also many places marked but without names. Whether these are all of the missing places from the copper plate edition is yet to be determined.

The map is part of a valuable atlas and great care is needed to digitise it. The page was carefully opened and straightened, photographed at 300 dpi and saved as a tif file. Unfortunately there are still some rather strong folds in the photograph but, as with the earlier maps described in Jupp (2017), the methods used here can overcome many (but not all) of the problems with folds. The result of fitting a Sinusoidal model to the d'Anville map, with Beijing as the central meridian at 116.39^{o}E is given in **Table 4**.

The parameter λ_0 (Eq 3) is again the standard (or central) meridian for the Sinusoidal projection. In this case it is not a fitted parameter but fixed at the longitude of Beijing. As with other maps digitized using photography, the nominal dpi of 300 is used to determine the map scale. The resulting principal map scale is about 1:1,100,000 which means it is enlarged further in relation to the copperplate map which was enlarged in relation to the woodblock maps. Because the ratio h_x / h_y is close to 1 it seems there is little difference in *x* and *y* pixel map scales. A map *RMS* of about 0.2 cm on the map sheet is rather large due to the folds. However, when the projection parameters are used to geocode the map and it is displayed in GoogleEarth the du Halde gazetteer points are found to fit at the places they represent very closely. The principal scales used by d'Anville seem to vary from province to province, but if it were desired to mosaic the d'Anville provinces, this could be overcome during re-projection, or by resampling if the basic projection is not changed.

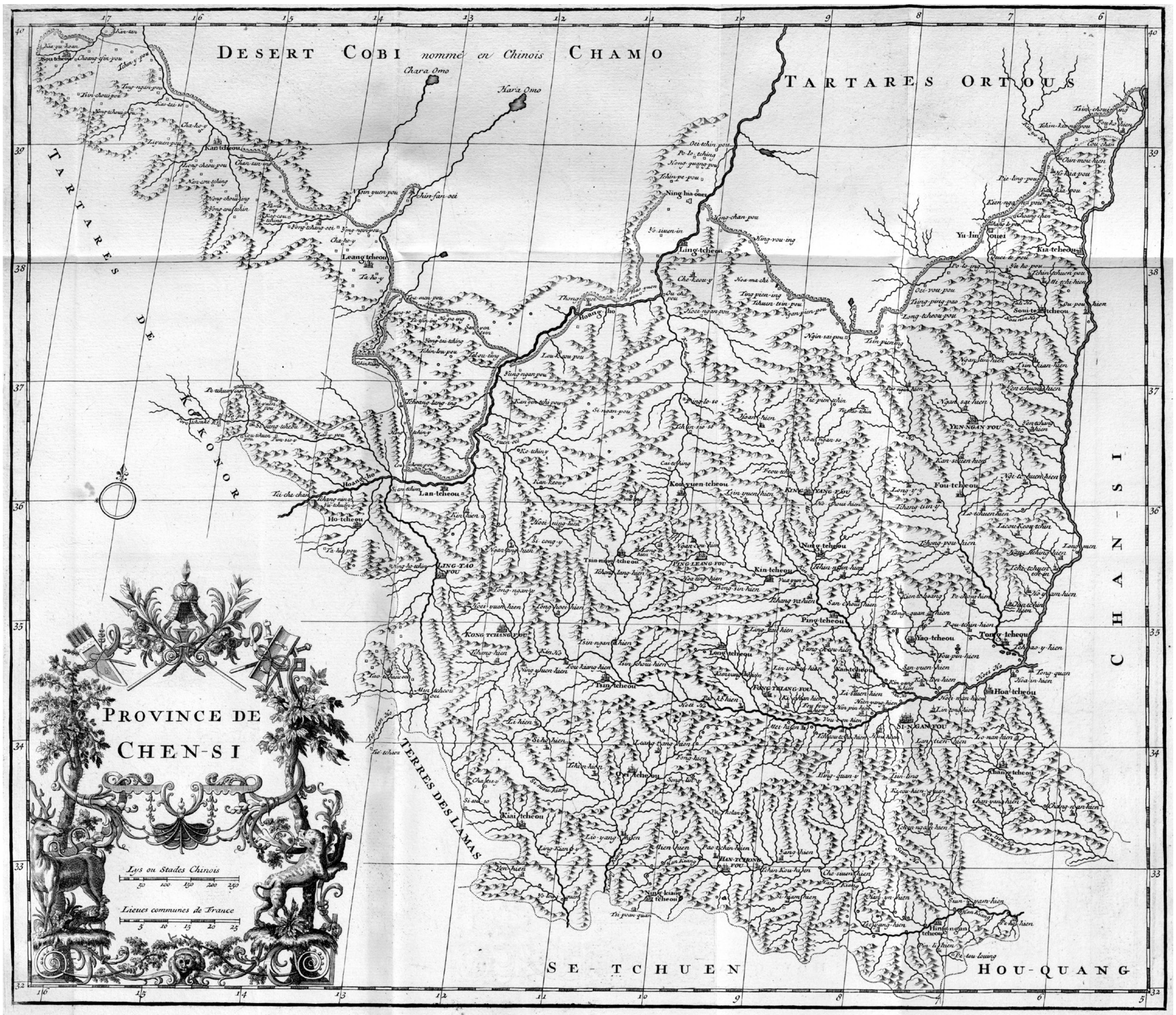

Figure 6. J.B.B. d'Anville (1785) *Province de Chen-si* [Shaanxi] from *Nouvel Atlas de la Chine* (National Library of Australia MAP RA 282 plate 12, online at http://nla.gov.au/nla.obj-658467733)

Table 4. d'Anville map of Chen-si [Shaanxi]: result of fitting a Sinusoidal projection

Parameter	Value	Units
x_0	-1,558,188.68	m
y_0	4,479,832.85	m
h_x	93.96	m
h_y	-92.40	m
λ_0	116.39	degrees
X_{err}	1.386	km

Parameter	Value	Units
Y_{err}	2.991	km
Total	2.189	km
Av pixel	93.18	m
Map scale = 1:	1,100,515	
Ratio h_x / h_y	1.0168	
DPI	300.00	
Map RMS	0.19889	cm

The 1721 Kangxi Jesuit map of Shensi [Shaanxi] province

It is also useful to compare d'Anville's 'Chen-Si' [Shaanxi] map (**Fig. 6.**) with that from the 1721 woodblock printed set from an edition printed in Beijing in 1941 (Regis *et al*. 1941) and used in Jupp (2017). Du Halde (1735) titled this map 'Shen Si' [Shaanxi] (**Fig. 7.**). It was most likely a part of the original mosaic from which the copper plate map was drawn. The 1721 maps contain a large amount of detail in places, including rivers and terrain, but were cluttered, and the characters were clearly drawn by artisans rather than scholars, and apparently at different times and probably by different people. The d'Anville maps, like the copperplate map, were obviously much more elegantly drawn.

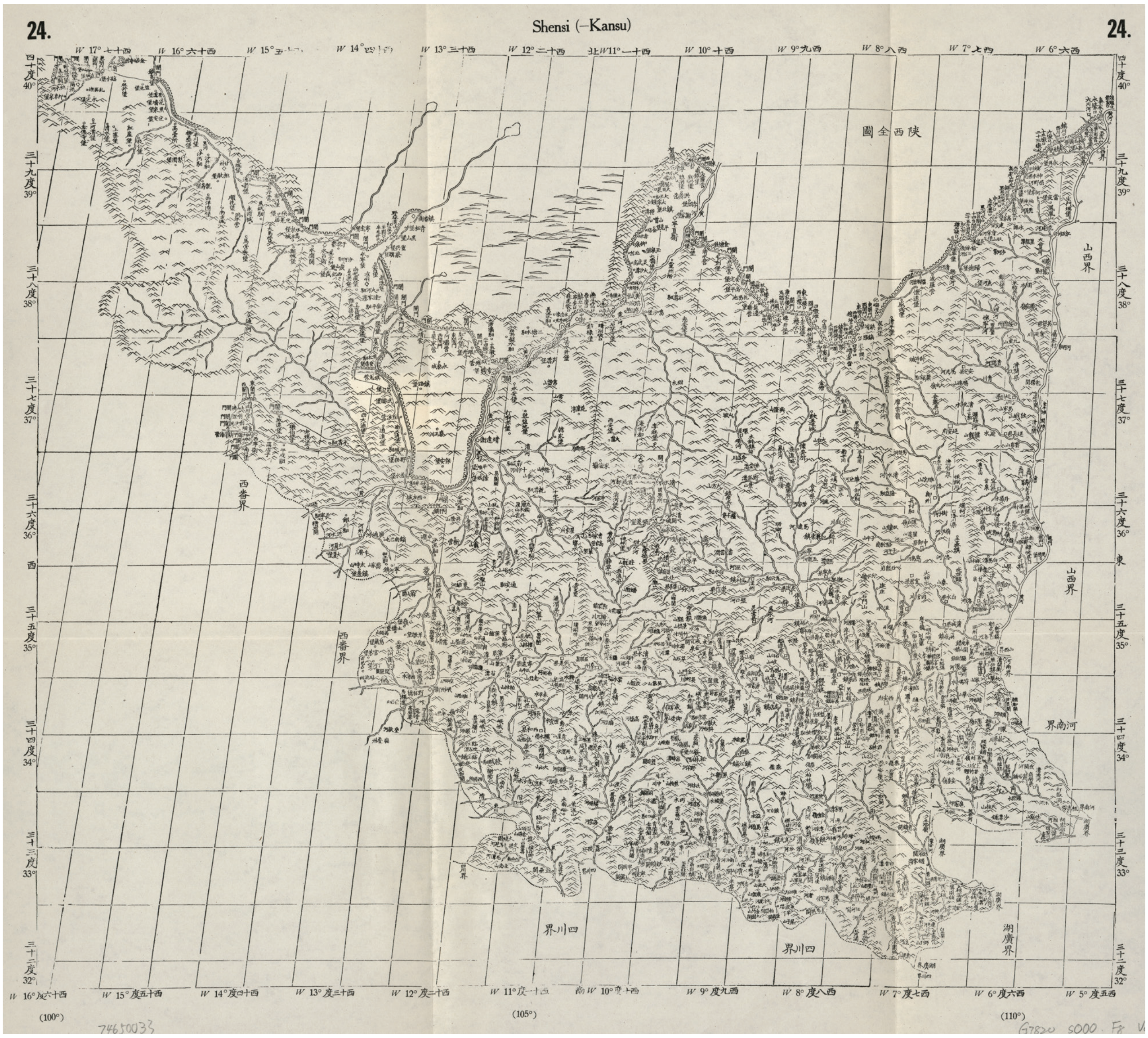

Figure 7. 1721 Kangxi Jesuit map *Shensi(-Kansu)*, i.e. Shaanxi province, from Regis *et al*. (1941)
(Library of Congress, online at www.loc.gov/resource/g7820m.gct00265/?sp=24)

Google Earth can be used to compare the maps and check consistency between the two ends of the chain of production. A study of the maps addressing how well the gazetteer points plot on the maps and also putting Chinese names to the points (only some of which have transliterated names) in the d'Anville map is not in the scope of this paper but is left for another time. Using the Martini, 1721 Kangxi, and d'Anville maps together, along with the available gazetteers, can also help identify places, despite the varying transliterations, and establish the historical name in Chinese characters.

The equivalent projection fitting results for the 1721 map are given in **Table 5**.

Table 5. Kangxi 1721 Shensi [Shaanxi] map: result of fitting a Sinusoidal projection

Parameter	Value	Units
x_0	-1,544,166.87	m
y_0	4,461,675.69	m
h_x	122.97	m
h_y	-122.95	m
λ_0	116.39	degrees
X_{err}	0.984	km

Parameter	Value	Units
Y_{err}	1.230	km
Total	1.107	km
Av pixel	122.96	m
Map scale = 1:	1,936,356	
Ratio h_x / h_y	1.0002	
DPI	400.00	
Map RMS	0.05719	cm

The parameter λ_0 (Eq 3) is once more the central meridian for the Sinusoidal projection at Beijing. The map was scanned by the Library of Congress at 400 dpi. Careful measurements of the map borders confirmed the accuracy of the nominal dpi. It was then used to compute the principal map scale. The principal scale here is about 1:1,940,000 so it is reduced in size compared with the d'Anville map with a greater consequent clutter of annotation. The plotting error of 0.06 cm is, however, clearly better than the d'Anville map. But both are similar in overall drawing accuracy in kilometres on the projection plane. Because the expansion of the principal map scale seems to expand the plotting error as well, it is consistent with the map being copied rather than the du Halde gazetteer points being re-plotted to guide the new map. The consequences of this interesting possibility can be pursued in another study.

DISCUSSION

Historical factors in the application of the methods

At the time these maps were produced (1650-1750) the modern mathematical approach to map projections was still to appear (Snyder 1993). This change occurred in the second half of the 18th century. At the earlier time, maps and their projections were still heuristically defined and manually drafted. They were also still produced by variants of the standard tools of Euclidean geometry: pencil, ruler, set square, protractor, divider and compass. Map projections were therefore for the most part based on circular arcs and straight lines and drafting accuracy was a mark of the best map makers. The globular and Nicolosi two-hemisphere world maps of Jesuits such as Matteo Ricci and Ferdinand Verbiest used only arcs and lines as did the earlier Conic, Trapezoidal and Mercator maps. An exception was the Sinusoidal projection which had curving arcs but was (at the time) most likely to have been approximated with piecewise straight lines. Principal map scale was a primary factor but the mathematical theory of map distortions and desirability of properties such as conformal, equal angle and equal area were concerns of the future. After the map was drafted, the use of manual re-projection and scale change, and the effects of older printing technologies – not to mention storage – took their toll. As a consequence of all of these factors the goodness of fit of the mathematical projections as measured using the graticule, and the importance of issues such as plotting accuracy of the Gazetteer points and using tie points to bind mosaics, are much more significant with early maps than they are for modern maps.

On the other hand, the more limited number of projections commonly used at the time makes the problem of selecting a candidate projection easier to solve. Wang (1991) used manual measurements to show, beyond doubt, that the copperplate maps the Jesuits drew from the Kangxi Emperor's survey used the curving equal area Sinusoidal (or Sanson) projection rather than the ruled Trapezoidal or any other projection. From this and similar papers it is also clear that once the projection was identified, parameters of the mathematical projection can also be accurately derived from the early maps. The Sinusoidal projection had been used earlier but it was especially promoted by Nicolas Sanson d'Abbeville (1600-1667) when he was France's first Royal Cartographer during the period around 1650.

It was therefore natural also to look at Martini's provincial maps for use of this projection. In relation to the general maps, **Figs. 1 & 2** easily suggest that European cartographers were still using the simple Conic projection for regional maps in the mid-latitudes. In these two maps, the parallels certainly seem to be arcs of circles and the meridians to be radiating straight lines as expected in this projection. The Conic could certainly be accurately drawn with the instruments available at the time with arcs of circles produced by sweeps of a large and well-engineered compass. To establish the projection and parameters of a given modern map of unknown projection may have the advantage of a much more precise map to use, but the range of possible choices for the projection is now almost too great.

Opportunities to use map projections and image processing to investigate old maps

The projections map makers used, and the principal scales at which the map was printed, are useful and interesting information. However, the projection parameters can be used in many other ways to help analyse and compare the maps between atlases. One important action is to re-project and rescale the images to different formats and display them in presentation software such as Google Earth. To do this, the images are often converted into a Google Earth 'super-overlay' using software such as Klokan Map Tiler (Klokan technologies, n.d.). A super-overlay structure allows an image to be displayed at different levels of detail from general to particular scales so that only small amounts of data are needed at each stage. In the referenced software, the image is broken up into tiles defining different levels of detail so that one can 'zoom' and 'pan' and change map scale as desired, right down to the finest resolution of the image. The amount of data being used at any one time is small, so that the data can reside on a remote server or a cloud computer without risking signal delays. The 'black' areas outside of the original map area can also be made 'invisible' to provide a convenient display in Google Earth. Super-overlays of the maps in this paper can be accessed via the Kangxi Maps web page (n.d.).

For example, if the d'Anville map of China (**Fig. 2**) is treated in this way it can be viewed as shown in **Fig. 8** and analysed using the available software:

Figure 8. Screenshot of Google Earth display of d'Anville's 1737 *Carte la plus generale et qui comprend la Chine…* .

An obvious error in **Fig. 8** is that at the north-west corner it has failed to overlay the Caspian Sea. When d'Anville's map was first released, it was criticized and also (apparently) improved in an alternative map by Englishman John Green (an alias for one Bradock Mead, according to Cams 2014, quoting others). Green claimed that the town of Astrakhan at the top left of the image was in error in longitude by about 3º. In fact, it is 6º east of the true position. The town of Astrakhan was the major 'anchor point' for the information in this region so the error was very significant. The latitude, however, is almost exact. Green suggested that d'Anville moved Astrakhan to match the mapping made by the Jesuits and mentions the error in its position relative to the Kashgar area as evidence. The Google Earth image can also be used to establish that Kashgar is 3º east of its true position, although its latitude is again very close. It seems Green was possibly right that d'Anville was juggling the different map components. However, if the lists of points in the gazetteer in du Halde's book are examined it can be seen that the Jesuits did not go to Kashgar, and that the furthest west they went in their survey was to Hami. Interrogating the image shows the Hami location is only about 20 km from its true position, and the error is basically in longitude. So the problems seem to be only in areas outside of the control of the Jesuit ground surveys.

Similar comparisons of different maps and gazetteers, as well as investigations of map accuracy, can be made when the maps become geocoded images. The places in the Martini and du Halde gazetteers can be easily plotted on the images to check the plotting accuracy of the original map, and the extent of map control. For the province maps of both Martini and d'Anville, establishing the projections also holds out the promise of creating mosaics, using computer technology, at the level of detail of the base province maps. Jupp (2017) constructed such a mosaic from some of the Kangxi 1721 maps and in recent (unpublished) work, a mosaic for the 15 provinces has been constructed using similar methods for the base province maps developed by the Jesuits. A draft version of this mosaic can also be found at the Kangxi maps web page (n.d.) where all of the images discussed in this paper can be accessed in super-overlay form.

As a final comment, some investigators may also wish their images to be 'warped' to fit modern maps (Klokan Technologies, n.d.) in order to investigate changes in historical boundaries and place names over the years. This has been done with many of the maps from the David Rumsey Historical Map Collection (David Rumsey maps, n.d.). To do this most effectively it is best to first transform the map so that there are minimal distortions to annotations and map information in the process. The methods outlined here can therefore also act as an effective pre-processing when warping is desired.

CONCLUSIONS

The National Library of Australia has a very good collection of well-preserved European maps of China from the period 1650-1750. In particular, the set of maps and atlases associated with Martini (1655) and d'Anville (1735, 1785) represent a significant flow of geographic information to Europe from China that came through the efforts of the Jesuit missions of the time. The d'Anville maps were based on the Kangxi period survey described by du Halde (1735) and placed in historical context by Yee (1994). Various publications including Wang (1991) and Jupp (2017) have shown how the Kangxi period maps used the Sinusoidal projection and can be treated as if they were modern maps to estimate principal scale and map accuracy. They can also be re-projected and mosaicked using GIS technology. This paper suggests that similar methods can certainly be used with the wider set of European maps and atlases being published at this time.

It has been shown that the projection and principal scale of the maps in the Martini and d'Anville collections can be identified, and that this can certainly lead to successful re-projection, changes in map scale, accuracy assessment and mosaics into new maps in new projections. In the future, comparative studies based on this approach can easily be carried out. For example, the various map collections can be used to identify places in both Chinese and transliterated forms over the historical period of the late Ming to early Qing that are common to the various maps. The methods were successfully applied here to a limited number of examples in order to encourage further studies and use of the technology in this situation. This was, however, only possible because the collections were well-preserved and managed, digitised carefully at high resolution and made accessible via web technology. This aspect of the work has been described in more detail by Dr Martin Woods in Woods (n.d.) in relation to the map collection of the National Library of Australia.

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