

# Scale, accuracy and mosaics based on the Jesuit Kangxi “Secret” Maps of 1721: Project Background and Details.

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## Summary:

The Kangxi Emperor employed Jesuit mathematicians (1708-1718) to design and manage the production of a set of maps of the provinces of China using a combination of western and Chinese survey methods. The maps were completed and the first maps presented to the Emperor in 1718 and finally in an updated form in 1721. The maps remained secret in China and were not used outside the court until updates were made in the late 19th Century. The maps were, however, quickly sent back to Europe by the Jesuits and became the basis for maps of China produced by Jean Baptiste Bourguignon d'Anville in 1735. The main change from traditional Chinese mapping was to use Latitude and Longitude as the primary coordinates and use astronomical measurements of latitude and longitude at selected places to establish baselines. Changes in latitude and longitude between other places was found by first using traditional metric survey to obtain distances and then using relationships between distance north and south and latitude and distance east and west and longitude to convert distances to degrees. The relationships between distance and change in latitude were established by survey measurements and based on a spherical earth model. A re-printed version of an early set of maps taken back to Europe was accessed from the Digital Library of the US Library of Congress. In this QinShuRoads web site Project, the digitised images were used to reconstruct the parameters of the original sinusoidal projection and then re-project the maps into Geographical coordinates for presentation and view in Google Earth. The work has been reported in a paper by David Jupp entitled “Projection, scale, and accuracy in the 1721 Kangxi maps” and has been published in *Cartographica* [0]. This document is a detailed collation of the material accessed and the image processing undertaken for this Project and is provided to support the published paper with additional material. It provides more detailed information about the accuracy of the 5 Provinces maps and the creation of mosaics and additionally compares the maps with those developed by Martino Martini 50 or so years before. The work is supported by a website and a Project page that can be accessed [HERE](#).

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# 1. Introduction

In 1718, on the basis of extensive field surveys covering almost all of the then Qing Empire, a group of French Jesuit Brothers and Chinese surveyors produced a set of province and region maps for the Kangxi Emperor. The maps were the most accurate cartographic mapping of China ever carried out before that time and were likely at least similar in accuracy to European maps of the time<sup>1</sup>. If they were not of equal quality to the European maps, the reasons would only have been the scale of the enterprise and the speed with which it was done (10 years 1708 to 1717).



Figure 1: Combined mosaic of the 34 Kangxi Jesuit maps sent to Europe

Chinese in general knew little about these maps until the 20<sup>th</sup> century. However, the information became available in Europe quite soon after 1718 and formed the basis for Jean Baptiste Bourguignon d'Anville's "*Nouvel atlas de la Chine, de la Tartarie Chinoise et du Thibet*", or "New atlas of China, Chinese Tartary and Tibet" [1] published in 1737 as well as a number of other "pirated" versions of the maps. Most of the maps of inner China at this time were reproduced by d'Anville almost without change as described by Mario Cams [2] in his informative paper. A mosaic of all the maps was apparently assembled by d'Anville and published as part of a monumental "encyclopaedia" of China as known from the missionary work of the Society of Jesus by du Halde [3]. Figure 1 shows one of the "pirated" versions of the assembled d'Anville map. A comprehensive contemporary description of the mapping activity can be found in du Halde's book for which French [3] and English [4] versions are available.

<sup>1</sup> In the east, Korea and the ocean side of Taiwan were not mapped in the Jesuit surveys. The substituted Korean map has significant longitude distortion. Only the China side of Taiwan is accurate.

The map of Figure 1 is at a broad scale and was re-projected (probably to conic) while retaining Beijing as prime meridian. The maps of individual provinces and regions developed by the Jesuits were on the sinusoidal, also called Sanson (*ca.* 1650), map projection with Beijing as standard meridian. The Kangxi maps drafted in China all used this simple and mathematically well founded projection although it is only recently that it has become fully accepted as sinusoidal. For example, Prof. Cordell Yee [5] in his comprehensive modern discussion of the Kangxi maps refers to the projection as “trapezoidal” whereas even at the time, du Halde [3] and d’Anville [1] referred to the projection simply as a “general” map projection (“*une projection Générale*”, du Halde, Preface p. lvj). Edward Cave [4] complained about the projection (which he translates as the “plain” projection) as “Being exhibited on the plain Projection with inclining Meridians, Countries are thrown out of their natural Figure and Proportion: Whence this Deformity, tho’ scarce discernible in the Maps of Pe che li, Shan tong, Kyang nan and Kyang si, thro’ which the Meridian of Peking passes, is yet very perceptible in those of Shensi, Se chwen and Yun nan, which lye farthest from it”. However, in a comprehensive study published in 1991, Wang Qianjin [6] showed that the projection is definitely the well-defined sinusoidal<sup>2</sup>. The maps he used were the copper plate edition<sup>3</sup> usually referred to (in English) as “Complete map of the imperial domain”. A set of the plates was found in the Imperial Palace at Shenyang in 1921. Because it was until that time unknown, it has since also been known as the “Kangxi Secret Map”. There are 41 extent plates with each plate being 39.7cm long and 67cm wide and when put together they form a complete map of China<sup>4</sup>. It has an inscribed projected grid with 1 degree spacing. Using the copper plates, because there were no paper stretch or fold distortions, the geometry was true to the original allowing Wang Qianjin to reach his conclusions on the basis of precise measurements. Wang Qianjin concluded that the scale of the copper plate map was 1:1.4M.

In 1949, a German missionary, Walter Fuchs [7], published facsimile prints of an original 1721 wood block printed set of 35 Province and region maps essentially identical to the maps later sent and published in Europe by d’Anville [1] as described by Cams [2]. In 1721 the maps also included updated information on Tibet compared with the earlier versions. A copy of these maps is held by the US Library of Congress where they have recently been scanned at high resolution into digital images [8]. The grid has 0.5 degree spacing and the maps include places down to township and garrison level. The present study has concluded that the maps are at 1:1.94M scale which makes them a little coarser than the map scale quoted for the copper plate maps<sup>5</sup>.

Prof. Cordell Yee [5] tackled the fairly complex story of the various early maps. An initial set of 28 Woodblock printed maps was apparently delivered in 1719, the copperplate version described above with 41 sheets was next and a revised version of 35 sheets appeared in 1721 and was sent to Europe by the Jesuit Brothers. The latter was almost certainly the version re-printed by Fuchs, scanned by the US Library of Congress and used here. Wang Qianjin [6] concluded that the Copperplate maps were produced at 1:1.4M scale and it will be shown later that the 1721 series were produced at near to 1:1.94M scale. Presumably the 1719 set

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<sup>2</sup> Later in this document it will be seen that the maps developed for China by Martino Martini (SJ) and published in Europe in 1655 were also in the sinusoidal projection and must have been contemporary with Sanson.

<sup>3</sup> Engraved by Jesuit Brother Matteo Ripa.

<sup>4</sup> It does not seem clear whether the d’Anville mosaic map in Figure 1 was based on the copperplate (1719) mosaic presented to the Emperor or on a new mosaic made by d’Anville of the 35 maps of 1721.

<sup>5</sup> The spacing of a 1 degree Latitude on the copperplates was measured by Wang as 7.95cm whereas the spacing of 1 degree Latitudes in the 1721 maps can be found to be about 5.75cm. So the copper plate map was enlarged relative to the 1721 maps. It was also a mosaic of all China cut into blocks rather than separate Provinces.

was also at about 1:2M scale as the 1721 set was an update of the initial set. Prof. Yee also suggests [5] that the original maps were drawn at 1:400k or 1:500k scale. However, this is more likely to have been the scale of the working survey maps depicting the survey lines and places along the way<sup>6</sup>. The final maps above would have been scaled to the final projection and included significant additional information based on local Chinese maps.

During the reign of the Kangxi Emperor, the Qing forces overcame the residual Ming resistance in Southern China (mainly Yunnan and Taiwan) and consolidated their authority north of the Great Wall. China also negotiated boundaries with Russia in the north. As described in some detail by Elman [9], the Jesuit provenance of the Kangxi Maps played an important role in the negotiations<sup>7</sup>. In 1708, as can be found in the “Historical Atlas of China” by Tan Qixiang [10], the Provinces and their boundaries in the maps seem much as they had been at the end of the Ming. What are today’s Gansu, Shaanxi and Ningxia were a single province and this Province (called ShanGan in the following), together with Shanxi and the Bei Zhili (present day Hebei) all have the Ming Great Wall as their northern boundary. The western boundary of Sichuan is to the East of its present situation and that of ShanGan generally to the East but includes Xining in the Northwest. Hubei and Hunan are combined as Huguang. Among other Provinces, the former Nan Zhili of the Ming is still present but is now called Jiangnan (江南) and its capital (present day Nanjing) is called Jiangning Fu (江寧府). The maps therefore provide a very interesting snapshot of a transitional stage between Ming and Qing regional government structures.

In this document, the five individual province maps of ShanGan, Shanxi, Henan, Sichuan and Huguang from among the 35 are analysed in detail for scale and accuracy. The specific projection parameters for the five provincial maps were established based on grid crossings and equality of borders and used to reproject the Jesuit maps to suitable form for import to Google Earth and also create a mosaic from parts of the Provinces. The mosaic was created for use in another study of traditional maps drawn between 1813 and 1822. The individual Kangxi maps were not warped using information from modern maps as was done by Alexander Akin and David Mumford in [9a] but rather reconstructed in their original form. This document evaluates the accuracy of the original maps and the mosaic in terms of the modern map information. The mosaic produced has sufficient accuracy and detail for its intended purpose. This document also describes findings of wider value to the study of the Jesuit mapping activity and the Kangxi Maps.

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<sup>6</sup> For example, the map brought back from the Great Wall survey was quoted by du Halde to be more than 15 foot (4.6m) long and the maps of Shanxi and Shaanxi were each said to be 10 feet (3m) square. Using information derived in this document, these represent about 1:600k scale or one third of the scale of the 1721 maps.

<sup>7</sup> Not only maps. Jesuit Brothers educated Chinese interpreters in Latin as the medium for communication with Russians. No doubt it also had other values to the Jesuit cause.

## 2. The Background and technology for the Jesuit survey of China

### *Summary of the activities of the Jesuit Brothers*

Most of the information summarised below concerning the Jesuit survey relevant to the studies made here has been gleaned from du Halde [3,4], Yee [5], Cams [2] and Han Qi [11]. The proposal to map the complete extent of the Empire apparently arose after successful field trials of the survey methods and efforts had been made to standardise survey measurements. The work plan was based on teams consisting of relatively few Jesuit Brothers, mainly French mathematicians, working with Chinese surveyors to survey transects throughout the country. The complete project occurred in roughly three stages being the development of the Great Wall as a baseline transect, mapping throughout the far north (Tartary) to help establish boundaries with Russia and finally the mapping of the 15 main “inner” Provinces of China. Maps of Korea and Tibet were included in the map set but were not mapped under direction of the Jesuit Brothers.

A set of reprojected images of the five provinces used here and the mosaics made from them have been developed as Google Earth super-overlays and are available at the web link [12]. In the images provided and in others available at [8], it is clear that the **Great Wall**<sup>8</sup> is mapped in considerable detail from Jiayu Guan across **ShanGan**, **Shanxi** and the Bei Zhili. It includes gates, forts and associated administrations along its full extent. In later surveys in northern China, including **ShanGan** and **Shanxi**, the Great Wall acted as a horizontal baseline. In du Halde [3,4] it is recorded that the Great Wall was mapped between July 1708 and January 1709 and du Halde [4] continues:

*“The 8<sup>th</sup> of May 1709, the Fathers Regis, Jartoux and Fridelli a German, Whom the Emperor had joined with them, set out from Peking to begin the Geography of Eastern Tartary, which is properly the Country of the Manchus who at present have the Dominion in China.....”*

The area north extended from the 40<sup>th</sup> to the 45<sup>th</sup> parallel (with measure places extending up to 51 degrees north) and relied on the baseline of the Great Wall. As described by Elman [9], the Russians had for some time been active on China’s northern boundaries and accurate surveys (based on maps Russians would accept) were needed to negotiate borders. After the Brothers arrived back in Beijing they were tasked to complete a map of the direct rule area around Beijing, or the Bei Zhili (北直隸, called Pe tche li by du Halde [3]). Such an important place obviously needed special attention and the work was only completed in June 1710. They were then send back north and North West up to the western borders threatened by Russian advances at a Latitude above 47 degrees. This and the pending activity and compilation was completed by the end of 1710.

From 1711, the surveys were divided between different groups as the work spread south and west throughout “inner” China. In the first split, one team (Brothers Regis and Cardoso) was sent to map Shandong. The other team (Brothers Jartoux, Fridelli and Bonjour) travelled beyond the Great Wall into what is today called Xin Jiang as far as Hami then mapped back

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<sup>8</sup> In this section, **bolded** text draws attention to places and areas contained in the five Provincial map sheets analysed here.

though that vast area, re-entering China at the Northwestern Jiayu Guan entrance of the Great Wall. In 1712, the Emperor asked if some others who were skilled in the survey methods could be found to join the effort and an additional four Brothers were enlisted. The major actions were (**bolded** areas below indicate those specifically relevant to this document):

1. Fr. Cardoso went to Shanxi where a Fr. de Tartre was stationed and with local surveyors mapped the provinces of **ShanGan** and **Shanxi**. According to du Halde's description, each map brought back to Beijing was 10 Feet square<sup>9</sup>. The two Brothers then went south to map the Provinces of Jiangxi, Guangdong and Guangxi.
2. Brothers Regis, de Mailla and Henderer mapped through **Henan**, Jiangnan (present day Anhui and Jiangsu), Zhejiang and Fujian.
3. Brothers Fridelli and Bonjour were sent to map the provinces of **Sichuan** and Yunan. However, Fr. Bonjour died near the border with Burma and Fr. Fridelli also fell ill. Fr. Regis was later sent to complete the map of Yunnan. Fr. Fridelli had recovered and together they went on to map Guizhou and **Huguang**.

When considering the accuracy of the maps in a later section it is necessary to take account of the conditions that prevailed, the teams involved and the routes followed in the various mapping missions as described above. The complete set of surveys was finished by January 1717. The next year was apparently spent collating the data and developing the final products to present to the Emperor.

## ***Astronomical Measurements and the Method of Triangles***

In Europe before the time the survey was carried out in China, regional mapping was based on Latitude and Longitude, used celestial navigation and assumed a spherical earth. It suited seafaring well and the spread of ships from Europe across the world was a strong stimulus to the developments. Map projections were becoming better established and nations competed for better methods in parallel with competing for trade advantage. However, on land and sea the serious problem of estimating accurate Longitude remained essentially unsolved. European astronomers had, by the time when the mapping of China was proposed, developed Tables to support the latest astronomical measurements of Longitude [13,14]. The best methods were originally devised by Galileo and involved observations of the crossing times of the moons of Jupiter. It was this technology that most likely provided a significant advance for the Kangxi Maps over previous maps.

From du Halde's [3,4] descriptions it appears that a large number of places in China had measured using these methods soon after the French Jesuit Brothers arrived. However, the astronomical measurements were not always used during the intense surveys in "inner" China after 1710. The problem was that in the difficult conditions of the field surveys, they could be subject to large errors. Rather, some of the existing places where Jesuit astronomers had spent more time to obtain accurate Longitude data in the past were used for baseline control. During the surveys, astronomical measurements of Latitude were, however, often made. These were most likely based on the elevation of the Pole Star with checks using sun position at noon. As time was short and weather not always cooperative, these measurements were sometimes also not very accurate. However, combining survey methods and astronomical measurements likely improved the Latitude estimates significantly.

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<sup>9</sup> By contrast, the map sheet for ShanGan used in this study is approximately 55cm square.



The primary survey technique was based on traditional surveying with chain (distance), compass (direction) and staff (height). If the route from one place to another was carefully mapped in short stages estimating distances, directions and changes in altitude then, after adjustments to the data, the Theorem of Pythagoras could be used to resolve the length of the route “as the bird flies” (feinia, 飞鸟) into an incremental distance north or south and an incremental distance east or west between the places. If the distances involved were less than (maybe) 200 km as the bird flies, the earth can be assumed locally flat and Euclidean geometry prevailed. The change in latitude and the change in longitude can then be estimated the relationship between distance over the earth and change in angles in the north-south direction (change in Latitude) and in the east-west direction (change in Longitude) are known. Starting from a place where the Latitude and Longitude are known, the corresponding Latitudes and Longitudes of other places along the survey route can then (in principle) be calculated and adjusted when astronomical measurements were made. This survey method itself does not result in absolute Latitude and Longitude but rather in increments of Latitude and Longitude. So, apart from places on a direct survey line from Beijing, some independent astronomical measurements were essential as “anchor points”. Since Latitudes were measured regularly, it follows that if the surveys also included enough places where accurate Longitudes were measured or known then corrections could be made based on these and overall accuracy could be high.

Chinese surveyors already knew effective methods for plane survey to measure distance “as the bird flies” and the principles were presented in ancient times by Pei Xu (裴秀) (Jin Dynasty, 晋朝, 265-420 CE). Chinese mapping took no account of earth curvature but if the areas mapped were smaller than (say) 400km by 400km it was not a big issue. It is therefore likely that forming teams of competent Chinese to undertake the surveying was not a problem. As outlined by Han Qi [11], before the work started the Kangxi Emperor insisted Princes of the realm learn the methods used by the Jesuit Brothers. Han Qi [11] then describes the trials made by the court before the main mapping survey was commissioned. He quotes from a record of the instructions given to the Princes by the Kangxi Emperor:

*“On the whole, the method used is mostly geometrical triangulation. Although the name sanjiaoxing [三角形, triangle method] did not exist before, the mathematical method must always have it as its basis. For instance, the method of Gou-gu [Pythagorean theorem] is derived from triangle, and this method was passed down from ancient times. However it was not recorded in books. Therefore people do not know its origin”.*

The Emperor equates the Triangle method with the principles of Pei Xiu rather than present day survey triangulation. In doing this he spoke with authority as the Kangxi Emperor had published a treatise on the “Derivation of Triangles” in 1703 [24]. Therefore, because resolving distance as the bird flies into increments north-south and east-west was well known, it was only in the step from traditional surveying to the astronomical system of Latitude and Longitude and the spherical earth that the activity moved out of areas already familiar to Chinese. It does seem that the use of sightings to towers or other landmarks in nearby towns was also combined with triangulation to “close the triangle” and occasionally the surveys would pass the same place again for cross-checking. But ultimately, it was the resolved incremental distances as the bird flies that formed the base data. This was possibly the main reason that the Jesuit Brothers were able to map the whole of China in 10 years whereas it took about 70 years (1668-1744, [23]) using survey by baseline and dense triangulation for France to be mapped to similar detail.



## Standardising the Li

It is clear that one of the most critical needs for the survey was to establish the relationships between degrees of Latitude and Longitude and distance on the ground. To be sure of the measured distances on the ground it was also critical to standardise the measurements in Chinese units of distance and in terms of the measuring devices used in the field. All of these were addressed before the surveys began in experiments carried out in company with the Kangxi Emperor or Qing Princes. In Europe at the time, the figure of the Earth was still supposed to be a sphere and the Jesuit Brothers would have known that on a sphere, the distance across the surface on a meridian due north (or south) between parallels for a 1 degree change of Latitude is everywhere the same and can be written as:

$$h_y = R \times \frac{\pi}{180}$$

In this equation,  $R$  is the radius of the earth and  $\pi/180$  is the angle of 1 degree in radians. For the spherical radius used later for the Sinusoidal Projection,  $R = 6371007$ <sup>10</sup>metres. It follows that the distance on this sphere is 111.1951 km. The Jesuit Brothers would also have known that for the distance on a parallel corresponding to a 1 degree change in Longitude, the formula changes with Latitude and is:

$$h_x = \cos \phi \times h_y$$

In this equation,  $\phi$  is Latitude so that at the parallel of Beijing (taken here as 39.16667 degree North which at the centre of the Forbidden City) the distance is  $h_x = 85.2842$  km. But kilometres do not help Chinese measure the distances (nor the Jesuit Brothers at the time) so it was necessary to measure the distances in Chinese Li. But the Li has never been well standardised and has varied in length equivalent to modern western standards quite considerably both in time and place in China over 1000's of years. Even in 1917, when in theory China was moving to use standard western units, Sir Eric Teichman, travelling in Shaanxi as a Consular Officer, reported [15]:

*“From the railhead at Kuanyint’ang to T’ungkuan is called a distance of 280 li, and can be done in three days. As far as Lingpao the li are “short li”; thence to T’ungkuan they are “long li”; the difference is most marked, the short and the long li averaging about a quarter and a third of a mile respectively.”*

Later on when he was travelling south from Xi’an on an ancient road to Xing’an, or present day Ankang, he writes [15]:

*“From Hengk’ou to Hanyin is called 110 li, but the road is good and the li very short (at least four to the mile), so that with ponies and pack mules the distance can easily be done in a day. The Chinese li is commonly considered to be the equivalent of a third of a mile, which, however, is only the case in out of the way parts of Kansu. Elsewhere north of the Ch’inling Shan ten li to three miles may be taken as a fairly accurate average. In the upper Han valley,*

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<sup>10</sup> This is the radius of the authalic sphere corresponding to the WGS84 spheroid.

*as in Szechuan, the li averages about four to a mile, or even more in the mountains; the difference north and south of the Ch'inling Shan being probably due to the use of animal and coolie transport in those regions respectively."*

The length of a Li in metres corresponding to the situations Sir Eric mentions can be compared and summarised as in Table 1:

Table 1: Estimated lengths in km of various definitions of a Li

Description	Teichman	Distance (km)	Number
Short Li	quarter of a mile	0.40234	276.37
Long Li	third of a mile	0.53645	207.28
North of Qinling	10 Li to 3 Miles	0.48280	230.31
Han Valley	4 to a mile	0.40234	276.37

There are three different values here of metres in a Li, these are 402m, 483m and 536m (to nearest metre). The Han Valley Li and Short Li are the same. Teichman reports that in the mountains south of the Qinling, the Li is even shorter than the short Li of Table 1 (402m). Clearly, unless the survey team could ensure that the Li was a standard measure, the results in different places could vary widely just through the use of a different "Li". The equivalent number of the Li in 1 degree for the sphere used above is given in the last column of Table 1 headed "Number".

Using field experiments carried out by the court in the company of Jesuit Brothers, the distance north corresponding to one degree change in Latitude (which can be determined by observing the sun at noon and/or the Pole Star altitude at night) was established to be 200 Li exactly. How this came about is described by Han Qi [11] quoting the court official Li Guangdi as follows:

*"In the 10<sup>th</sup> month of Renzi [Wu] year [1702], His Majesty arrived at Dezhou during his Southern inspection. [...] As calendar experts described, one degree in the sky, [is] equivalent to 250 li on ground level. Although I have not surveyed precisely, I feel that the distance should be 250 li. At present I have asked San-a-ge [三阿哥, third child] to carefully measure the distance from Beijing. San-a-ge's mathematical skills are extremely refined. Now at Dezhou, albeit a little inclined to the East, Gou-gu method [勾股, i.e. the Pythagorean theorem] is used to measure, making use of pegs-and chunks to note the distance. Imprecise measurements will not happen any longer. Upon return to Beijing on the 21st, the Emperor said to [my] master: "San-a-ge has made the measurement, which means: one degree in the sky is exactly 200 li on ground level." My master said: "This is so because the system used was of eight Cun [寸 or Chinese inch] of Zhou dynasty's Chi [尺 or Chinese foot]<sup>11</sup>, resulting that 250 li equals one degree."*

Applying this to the distance above in km we find that the Li values corresponding to 200 and 250 Li to one degree on the sphere are 555.98m and 444.78m respectively. These seem to correspond quite well with Sir Eric Teichman's Long and Short Li. So the Emperor chose the Long Li which was then a new standard for the (Long) Li. The arguments based on historical records were most likely necessary for it to be accepted by the existing authorities.

<sup>11</sup> Traditionally, 1 Chi (尺) was 10 Cun (寸), 1 Bu (步) was 5 Chi or 6 Chi and 1 Li (里) was 300 Bu or 360 Bu giving various short and long Li's depending on choices.

The short and long Li certainly have an interesting correspondence with short and long definitions of the French League<sup>12</sup> at the same time. Corresponding to the Chi was the French Foot, or Pied, which was 0.3248 m or 1.066 Imperial Feet. Before the Revolution, the League had a number of definitions. Two had recently been related to the length of a great circle arc of 1 degree. There was the “League of 25 to a degree” at 2282 Toise (1 Toise being 6 Pied) and another established by the Paris Academy which Fr. Regis reported as being 2853 Toise for which there were 20 to a degree of a great circle arc. So the Short League is 10 Short Li and the Long League is 10 Long Li. It was no coincidence.

Despite Chinese explanations in terms of variations in traditional definitions of the relation between the Cun and Chi, what had happened was that a new standard had been created for the Li and used for the survey. Provided the Jesuit Brothers’ surveys used chains in the field to measure distances in terms of this (new standard) Long Li and any chains obtained locally were also calibrated to the standards, the survey had all that it needed and the Brothers could happily also convert into French Leagues. Fr. Regis called the French Long League the “Marine League”.

### 3. Map Projections and Mosaicking

#### *Province Map Sheets*

The base materials are the set of maps originally printed by Walter Fuchs [7] and held by the US Library of Congress [8]. In this document, five maps are used from the 15 “inner” provinces. They are called here by the names ShanGan, Shanxi, Henan, Sichuan and Huguang. Huguang was a Province comprising today’s Hubei and Hunan and ShanGan comprises present day Shaanxi, Gansu and Ningxia. The others are the present Province names.

Table 2: Five Provinces map sheets & Mosaic with Chinese and du Halde names and bounds

Map	Map Name	Chinese	Du Halde Name	Min_Lat	Max_Lat	Min_Lon	Max_Lon
24	Shensi (- Kansu)	陝西全圖	Chen Si	31.5000	40.0000	98.3910	111.8910
23	Shansi	山西全圖	Chan Si	34.5000	41.0000	109.8910	115.3910
25	Honan	河南全圖	Ho Nan	31.0000	37.5000	109.8910	116.8910
34	Szechuan	四川全圖	Se Tchuen	25.5000	33.0000	100.3910	110.3910
29	Hukuang (Hupei-Hunan)	湖廣全圖	Hou Quang	24.5000	33.5000	107.8910	116.3910
	Yan Ruyi Maps Mosaic			30.0540	35.7130	103.7790	112.5370

Table 2 summarises the Map Sheets in the order used here and includes the Map number, Name on Map (by Walter Fuchs), Chinese Name (Traditional characters), Name used by du

<sup>12</sup> A kilometre was not defined until after the (yet to happen) French Revolution.

Halde [3] and bounding box in four columns estimated by the present writer. The Table also defines the geographic limits for the planned Mosaic which uses data from the five Provinces.

The five maps were scanned consistently by the US Library of Congress at 400 dpi (dots per inch) and square with north at top. There were some local distortions due to paper stretch and folds being too strong for the sheet to lie completely flat. Small remaining rotations from north were removed in Photoshop and the frame trimmed to the neat line plus a margin to retain the latitude and longitude annotations. The image data (as TIFF files) were saved as grey scale and changed so that zero did not occur. Later, “zero” is used to represent null data and interpreted as “invisible” for mosaicking and display. The scan of the ShanGan Province in its native projection is included here as Figure 2 to indicate the typical base form.

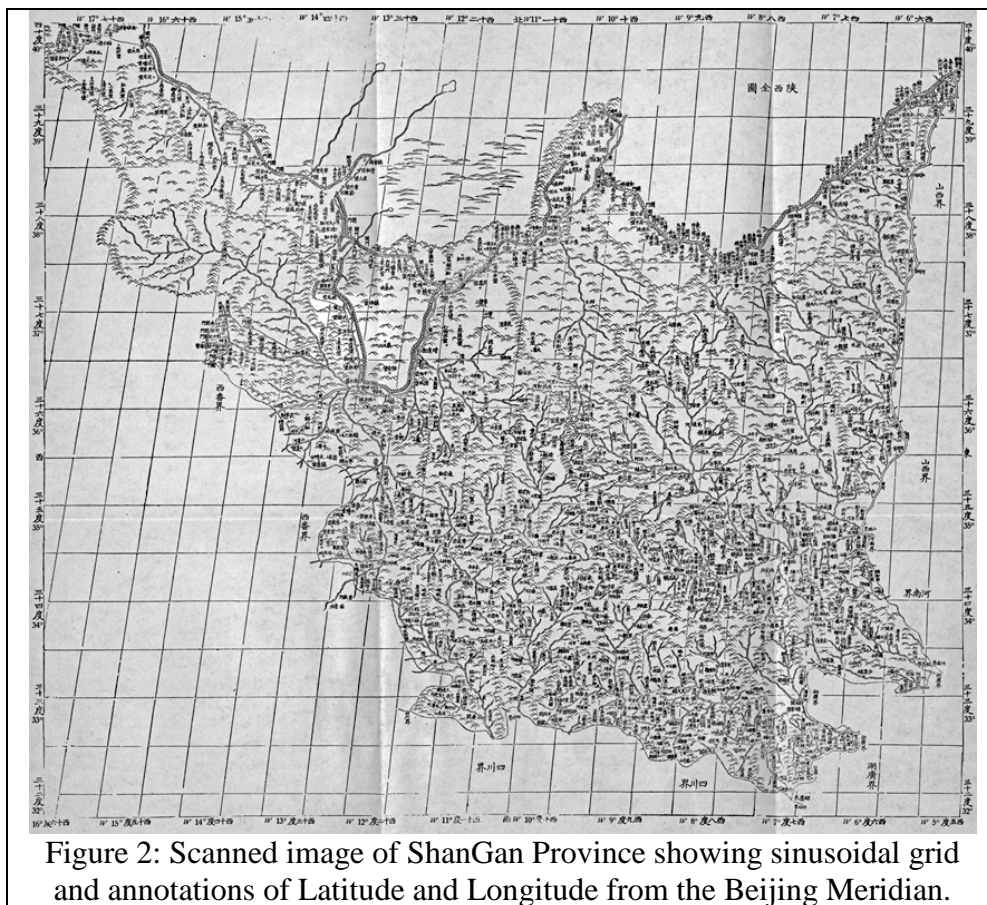


Figure 2: Scanned image of ShanGan Province showing sinusoidal grid and annotations of Latitude and Longitude from the Beijing Meridian.

The effects of the map folds are clear in this image. The grid lines are latitude and longitude at 0.5 degrees spacing with longitude as difference from the Beijing meridian. Using measurements of page and neat line sizes made by the US Library of Congress (Ed Redmond, Geography & Map Reference Specialist, Library of Congress; Personal Communication) it was confirmed that the scans were close to precisely 400 dpi in both X (across the image) and Y (down the image) directions. This means that the spacing of the dots (that become image pixels) on the page was 0.00635 cm and the dots had equal size in X and Y. This can be taken as the size of a pixel on the original page and will be used to determine the scale of the maps.

## Control Points

In du Halde's book [3] he provides a Gazetteer with a separate Table for each mapping region (eg Provinces) listing altogether more than 300 places where the basic ground survey and astronomical measurements were combined to calculate Latitude and Longitude. The places in the Gazetteer can also be used to assess the intrinsic accuracy of the mapping. Intrinsic accuracy is taken to be the size of differences between these places and known locations for the Qing Period places corresponding to the ones in the Gazetteer. As other places in the final map are most likely located by scaling information from existing local maps relative to the base plotting data, the direct total accuracy will be no better than this. The reference data set used here to estimate intrinsic accuracy was developed by the China Historical GIS (CHGIS) Project at the Fairbank Center for Chinese Studies [16] at Harvard University in cooperation with Fudan University in China. It will be referred to in the following document as the "ChinaW" dataset and comprises a collection of Qing Period places at District Level and above (Dao, Fu, Zhou, Xian and Ting) as they were in the period 1820-1893. A few of the places in du Halde's Gazetteer could not be found in the ChinaW set (as they were small forts or other places below Ting level) and were not included. The Longitude coordinates from Beijing were calculated assuming the Longitude of Beijing was 116.391 degrees East of Greenwich<sup>13</sup>. These points were saved as KML files and plotted in Google Earth. They are quite extensive and so have not been listed here but rather collected in a separate document to be made available on the web [12].

The Gazetteer in du Halde makes it clear that these 300 or so points are places where the Latitudes have been measured and where the Longitudes are the result of geometric measurements – meaning survey and the method of triangles. So it is not immediately obvious which were the places corresponding to the astronomical estimates of Longitude which are called "anchor points" in this document. The commentary by Fr. Jean-Baptiste Regis and quoted by du Halde [3,4] indicates that despite early enthusiasm for taking astronomical measurements they later became concerned that such observations would not be accurate enough due to the hard and somewhat rushed situation of their extended surveys. Regis wrote [4]:

*"... after mature deliberation we thought it best to [only] use the method of triangles, all others appearing to us to be not only too tedious, considering the vast extent of the countries of which the Emperor wanted the map, but scarcely practical on account of the towns being so near to each other; since it is certain that the least error, occasioned by the pendulum going wrong, or the immersion of one of Jupiter's satellites not being accurately observed, would cause a considerable error in the Longitude."*

The method of triangles is a survey method that is best used between places close enough for the flat earth geometry to apply. As the route continues, increments in Latitude and Longitude are calculated and accumulated. As Latitudes are measured at each place, there are some feed-back corrections that allow independent and relatively reliable Latitudes to be provided at the base locations. However, unless at least one reference place is included where absolute Longitude is known the value will always only be relative to the starting point of the survey. Some fixed points of reference (especially for Longitude) in other places are definitely

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<sup>13</sup> There is an apparent survey line through the middle of the Emperor's Palace and the Tian Tan Temple. But it is not quite a meridian. The point chosen is on this line half way between the reference points.

needed to maintain the baseline of the survey and peg the widely separated maps into an accurate framework. Because of this, Fr. Regis continues:

*“We therefore contented ourselves with observations of the Moon and satellites of Jupiter made before our time by members of our Society, though we rejected a few because they did not agree with our measures, on account of some small error as to time in the observation, which too often happens to the most experienced.”*

The English translation of du Halde’s book [4] by Edward Cave only lists the Fu level places in du Halde’s Gazetteer (on the map rather than in the text) and Cave claims that places with double underline in his combined map are those where longitude observations were made. There seems to be no information like this in du Halde’s book [3] but Edward Cave’s edition also references a book by Fr. Étienne Souciet [18] who collected ancillary information about astronomical observations in China and India at this time and so perhaps it is here that Cave obtained the places. Souciet’s Tables are (unfortunately) in Longitude from Paris but if his assumption for the Longitude of Peking from Paris is estimated (it is possible) the places in China can be located in the same form as du Halde’s Gazetteer. Again unfortunately, compared with du Halde’s Gazetteer, the Tables in Souciet [18] are messy and often inaccurate. Many observations are obviously not useable and at impossible locations. By finding places that are feasible and occur both in Souciet’s Tables and in the text (there are many discussions of observations in the text) as well as in du Halde’s Gazetteer, a set has been identified which lists possible locations where direct observations were made prior to the surveys for the Kangxi Map and could then have been used as anchor points for the surveys.<sup>14</sup>

Because it is only useful but not vital to identify these places as sources of astronomical measurements in this document, they have been collected in a Table and listed here as the Appendix 1. The basic relative accuracy of the points can be established using the Gazetteer provided by du Halde [3] and the information in the ChinaW data base [16]. However, when reasons for specific local or general distortions are discussed it will be useful if inaccurately measured Longitude anchor points can be identified when they are the cause.

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<sup>14</sup> It seems unlikely that Longitudes were observed by simultaneous observations at Beijing but rather calculated by using Ephemerides based on Paris to obtain Longitude from Paris. The results were certainly not very accurate as seen in Souciet [18] and the Longitude of Beijing probably used to convert to Longitude to Peking also seems inaccurate.

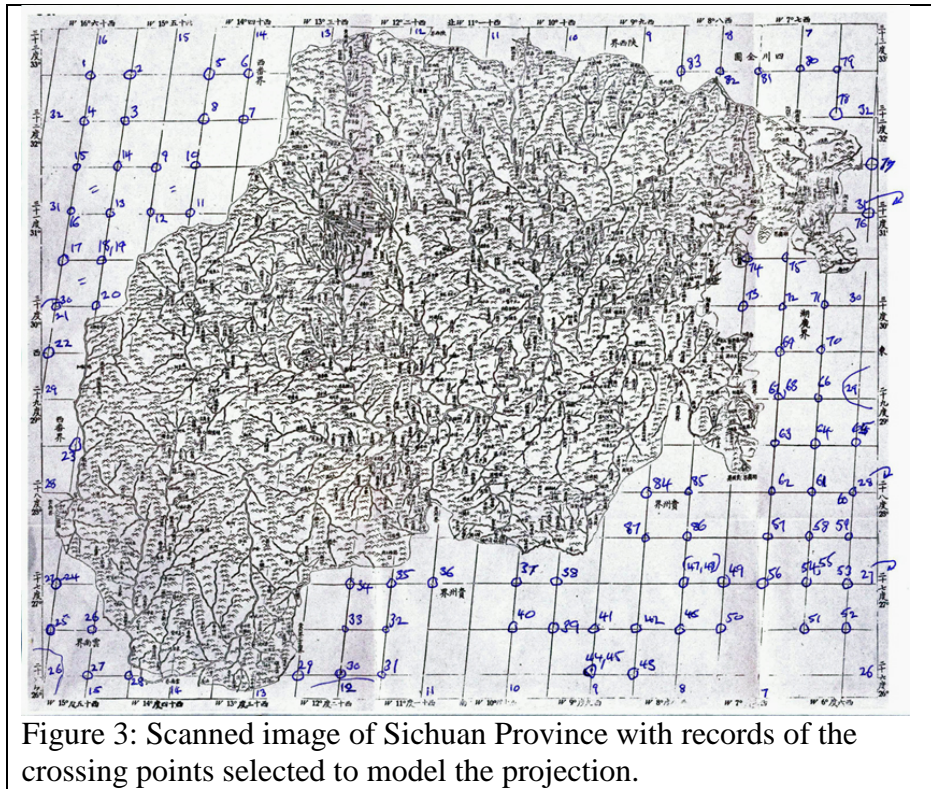


Figure 3: Scanned image of Sichuan Province with records of the crossing points selected to model the projection.

### ***Sampling the projection grid***

Figure 3 shows the scanned map for the Province of Sichuan. The edges have been trimmed so that only the map and the coordinates around the edge are visible. The first task is to identify grid points in the coordinate system of the scanned map and use them to calibrate the projection. The geographic coordinates are indicated by the Chinese characters with English added to the left on top and bottom and underneath on the two sides. On the top line of Figure 3, 九西 or W 9° indicates the Longitude is 9 degrees West of the Beijing Meridian. On the sides of the map, the Latitudes vary from 26 to 33 degrees North of the equator. As with other maps, the folds have produced distortions locally and the original block printing is also of variable quality – but names are mostly clear provided you can zoom to full resolution. Figure 3 also shows the set of crossing points selected and marked on the copy of the map as cross reference. The grid is not clear within the mapped area so there are none inside the province. To the West of Sichuan the annotation reads “西番界” or border with the western (Tibetan Ethnic) tribal areas.

To estimate map parameters, first, a relatively large number of the crossing points for the grid were located in the image and recorded in terms of the sample number (number of dots from the left hand margin) and line number (number of dots down from the top margin line). Samples were also collected using significant features on the common borders with the other map sheets. Each crossing point corresponds to a Latitude and Longitude as read off the sides of the map. Later, the base survey places are used to assess map accuracy. They could also be used to help fix the grid but were not needed in the maps used here.



## Calibrating the projection

Let Latitude be represented by  $\phi$  and Longitude by  $\lambda$ . A projection is a mapping with specific properties (especially 1-1, invertible and also differentiable) onto a Euclidean plane with coordinates  $(x, y)$  such that:

$$\begin{aligned}x &= f_x(\phi, \lambda) \\y &= f_y(\phi, \lambda)\end{aligned}$$

This allows given Latitude and Longitude values on maps to be converted into  $(x, y)$  coordinates on the projection plane and the inverse model allows points of the projection plane to be converted into equivalent values of Latitude and Longitude. The general theory of such projections is well covered in [18a]. After many years when the specific projection used by the Jesuit Brothers was unclear, the identification of the projection with a Sinusoidal Projection [18a] was proven unequivocally by Wang Qianjin [6]. Wang showed that the projection of the maps in the copperplate edition can only be the Sinusoidal. He uses two primary pieces of information, convergence of the meridians and the fact that parallels of Latitude are horizontal parallel lines and equally spaced. Equivalent to the convergence of the Meridians would be to note that the steps between intersections of Longitude on any one parallel of Latitude are equal with value of  $\cos \phi$  times the distance between the adjacent parallel Latitudes. Wang Qianjin showed their relationships by direct measurement. He also used his measurements to show that the scale of the Copper Plate maps (which had 1 degree spacing in Latitude and Longitude) is about 1:1.4m.

The projection equations for the Sinusoidal projection are:

$$\begin{aligned}x &= sR(\lambda - \lambda_0) \cos \phi \\y &= sR\phi\end{aligned}$$

In these expressions, apart from what is already been defined,  $\lambda_0$  is the reference longitude, R is the Radius of a Spherical earth and s is the scale factor. In the present case,  $\lambda_0$  is the Longitude of Beijing. The sinusoidal projection is based on a sphere. It is well known that that the form of the earth is better represented by a spheroid and among the common modern spheroids is the WGS84. It is consistent with GPS and most maps now take it as the base form. However, at the time the map was developed the earth was still regarded as a sphere. For the present work, the Authalic<sup>15</sup> Sphere for the WGS84 spheroid (radius=6371007m) has been used. If the scale factor is 1.0 then the units of the coordinates are metres on the sphere and if the coordinates are (eg) cm in the printed map page, “s” will represent the map scale that converts into that coordinate frame.

The figure of the whole earth in sinusoidal projection is shown in Figure 4.

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<sup>15</sup> The Authalic sphere for a spheroid is the sphere with the same centre and the same surface area.

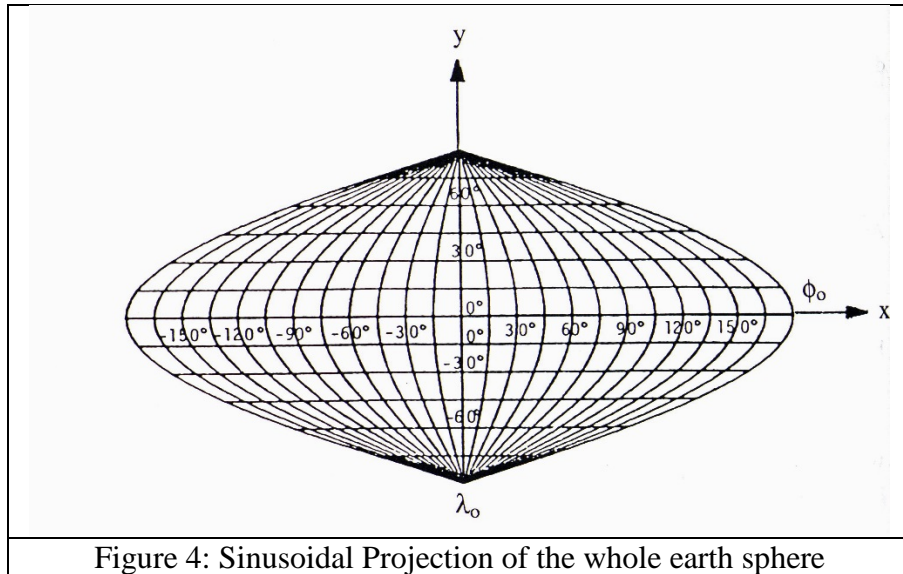


Figure 4: Sinusoidal Projection of the whole earth sphere

The zero of the (x,y) coordinate system in Figure 4 is at the crossing of the reference Longitude and the equator. As a whole earth projection it is very convenient. It has the nice property of being an equal area projection but distance and angle distortions become large away from the middle of the map. The sinusoidal projection was especially convenient and suitable for the mapping exercise carried out by the Jesuit Brothers. It precisely matches their survey model with 0.5 degree of Latitude north or south being a constant distance on the surface and parallels having the distance corresponding to 0.5 degree of Longitude east or west being  $\cos \phi$  times the distance between the parallels. The distance north 1 degree on the Authalic sphere is simply  $\pi R / 180$  or 111.1947 km.

This leads to a simple method to calibrate the projection for a given map. Supposing that the scanned grid of samples and lines (j,i) is a scaled grid on the Euclidean (x,y) plane, it is possible to model the coordinates as:

$$x = x_0 + h_x \times j$$

$$y = y_0 + h_y \times i$$

For the set of image sample and line values (j,i) for grid crossings there is a corresponding set of Latitudes and Longitudes ( $\phi, \lambda$ ) read from the map. These Latitudes and Longitudes can be converted into sinusoidal coordinates (x, y) in metres relative to a central meridian at Beijing. The factors ( $x_0, h_x, y_0, h_y$ ) which best calibrate the above equations to provide sinusoidal (x,y) can be computed through minimum RMS error. Errors will arise from errors plotting the grid, paper stretch, wood block printing errors or uncertainties due to paper folds. The 4 fitted parameters provide a model for the projection. In particular, the scale parameters ( $h_x, h_y$ ) indicate the scale of the maps.

The grid crossings tell you nothing about drawn map accuracy – just about the initial drawing and representation of the projection through the grid. With mosaics in mind, the common borders between provinces were sampled for specific features that were identified in adjacent maps. The constraint that these must be equal in the whole projection is quite a strong constraint and involves the drawn map data itself. These types of constraints were applied to

the five maps used here. The fit is made in the projected space and so errors can be expressed in metres. A summary of the results obtained in the five maps has been presented in Table 3.

Table 3: Summary of fitted Projection coordinates and errors if fit

	x0	y0	hx (m)	hy (m)	x_err (m)	y_err (m)
Shensi_nu	-1544752.00	4461094.77	123.08	-122.78	637.64	1115.93
Sichuan	-1575148.77	3685246.54	123.06	-123.20	2103.73	823.48
Huguang	-838352.05	3736215.42	123.66	-123.02	811.16	1240.87
Henan	-609718.94	4143776.82	122.43	-122.26	744.63	556.62
Shanxi	-602587.95	4568284.87	122.90	-122.39	525.74	625.95

The units in Table 3 are metres and it is clear that most average map sheet RMS errors are less than or of the order of 1km. Exceptions in Sichuan and Huguang were associated with fold distortions which were off the drawn map areas. These errors are much less than the 5-10km (and greater) errors we will see later for true place errors. The fits between shared sections of province borders are not shown separately but were much less than the grid point errors indicating good prospects for successful mosaics. The average pixel size is 122.897m giving the scale previously quoted to be about 1:2M or more precisely 1:1,937,000<sup>16</sup>. These parameters allow the images to be geo-referenced for a Sinusoidal projection and (if desired) changed to another projection. For Google Earth it is wise to change the projection to Geographic as in Figure 5. The grid is now parallel, equally spaced and normal in vertical and horizontal directions. Grid cell sizes are the same in each direction. Google Earth can drape this kind of image properly over the sphere.

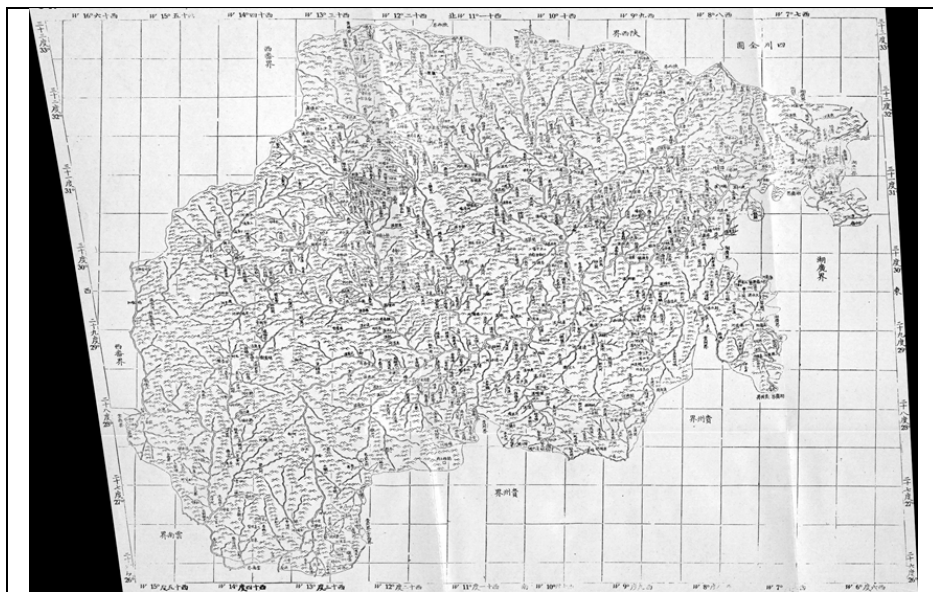


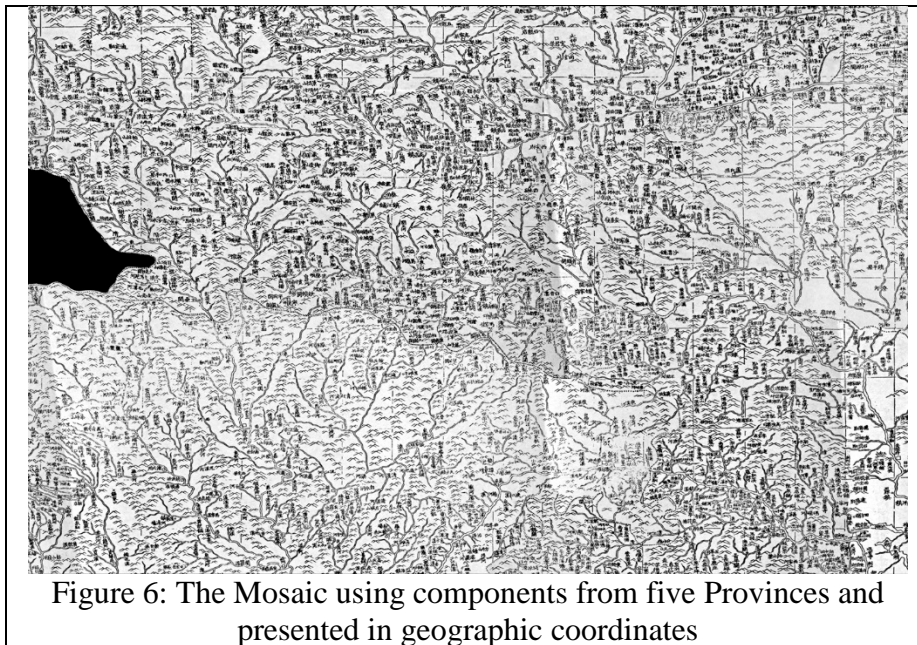
Figure 5: Sichuan Province re-projected to geographic projection showing new grid lines as orthogonal equal spaced parallel lines

<sup>16</sup> The scale factor (s) is  $s = \text{Scan\_dot\_size} / \text{pixel\_size}$  and scale is normally written as 1:1/s

## ***Image Mosaics***

Two mosaics have been created and saved as Google Earth super overlays. One was to the geographic coordinates given Table 2. Since a result was wanted specifically for this area, the images had been converted into a geographic projection for Google Earth overlay were cut to the part of the rectangular (in Latitude and Longitude) area previously defined. The boundaries were digitised and areas outside province boundaries were set to null (zero). This allowed the areas outside the boundary to be treated as if it were “transparent” when the mosaic comprising the final area was made ready to view. The Mosaic image is shown in Figure 6. The small remaining black (null) area is outside of Sichuan and Gansu and was not of interest in the target application of the mosaic. In the Google Earth super-overlay for the mosaic the null border is “invisible”.

Differences in contrast indicate different map sheets but due to the care taken with the projection model and the original accuracy of the drawing, the boundaries overlaid seamlessly. Once the parameters of the projection are known and the coordinates of pixels established, most image processing systems will allow the images to be combined with other geographic data or geographic information to be read off the image. They also allow the image to be re-projected into a new projection type. In this case, images had already been re-projected to geographic form to mosaic and in a later project the mosaic will be re-projected to transverse Mercator to match the square grid system common among local Chinese maps. Further information about the maps and access to images can be found at reference [12].



The second mosaic was to put all of the 5 Province maps into a single mosaic. The principles were the same as for the sub-area above except that the boundaries were fully digitised in a polygon around each Province and the mosaic was formed in the original sinusoidal projection. The boundaries matched well in all cases possibly due to the added constraints. Outside the boundaries the data was “null” (zero) allowing the software to see through to an image under a given Province image. Finally the mosaic was reprojected to geographical form and save as a Tiff file. The Google Earth super overlay software allows the outer “null” areas to be transparent to form a convenient image for presentation. The overlay can zoom up

to 7 levels so that at the finest detail the characters that are printed clearly can all be read. Figure 7 shows an example of the image view in Google Earth.

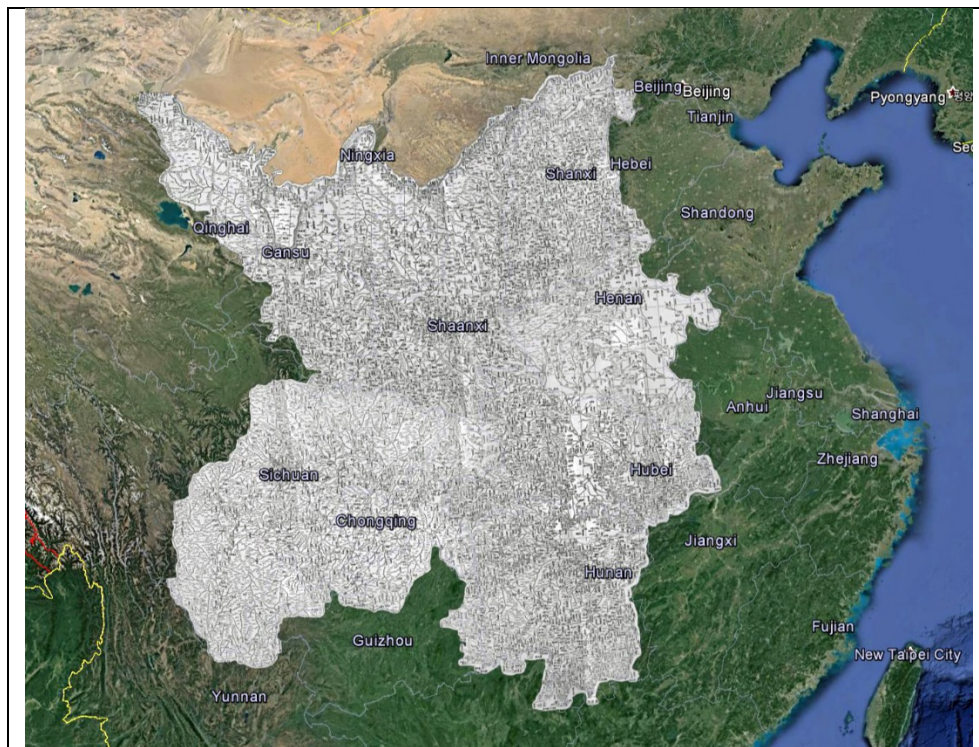


Figure 7: The Mosaic of five Provinces presented in geographic coordinates

## 4. Accuracy of the maps and the mosaic

### *Overall intrinsic accuracy*

The intrinsic error in the maps can be established using the base points in the du Halde [3] Gazetteer. Using 116.391 degrees East of Greenwich for the reference Longitude of Beijing, the Latitudes and Longitudes were available in equivalent coordinates. First, some of the plotting positions of the base points were checked manually on the maps and almost all were very close. This is expected if they were plotted correctly using the grid. Wang Qianjin [6] made a more detailed study of plotting positions for the copper plate map in the 15 “inner” Provinces of China. He found that there were occasional quite large errors which may be transcription errors in the positions in du Halde’s Gazetteer [3]. Wang Qianjin found errors in Longitude were greater than errors in Latitude which could represent the greater difficulty of drawing the Longitude grid accurately and of the manual interpolation – due to the convergence of parallels. But overall his conclusion is the same – they are mostly small and plotting is basically accurate. In the future, any possible transcription errors in the Gazetteer need to be checked and the plotting positions could usefully be included in the modelling of the parameters. But this has not been done in the present document.

Using the ChinaW set of Dao, Fu, Xian, Zhou and Ting places as they were in 1820-1893, the place names were identified and the coordinates matched with those in the du Halde



Gazetteer. In this case, the statistics measure true accuracy on the ground. Taken overall by Province, the errors (in units of kilometres) are listed in Table 4. The errors in Latitude and Longitude have been converted into km in the inverse way to how the Jesuit Brothers converted Li to degrees. The average error and standard deviation of error are both provided to identify possible regional bias error and RMS is the total error

$$(RMS = \sqrt{mean^2 + SDev^2}).$$

Table 4: Summary of errors (km) for five Provinces and Mosaic based on ChinaW data base

Short Name	Chinese Name	Du Halde Name	Av Lat Err	SD Lat Err	RMS	Av Lon Err	SD Lon Err	RMS
ShanGan	陝西	Chen Si	-0.800	9.024	9.059	1.660	21.325	21.389
Shanxi	山西	Chan Si	-3.530	8.465	9.172	9.318	7.689	12.081
Henan	河南	Ho Nan	-7.824	3.778	8.688	-1.561	10.228	10.346
Sichuan	四川	Se Tchuen	0.310	9.541	9.546	-0.047	5.033	5.033
Huguang	湖廣	Hou Quang	1.012	7.556	7.624	21.090	10.467	23.545
Mosaic			-1.016	8.222	8.285	4.948	8.968	10.242
Mean All			-2.166	7.673	7.973	6.092	10.948	12.529

These overall figures do not identify the locations of specific areas with significant errors but the basic message is clear. The overall RMS seems to be about 8km for Latitude and 12km for Longitude. ShanGan and Huguang seem to have significant Longitude errors and if they are left out, the overall average RMS drops to about 10km. Shanxi seems to have a bias error in both Latitude and Longitude and Henan a Latitude bias error. Huguang has a very large Longitude bias error. The mosaic area is as good as any other area with Standard Deviations of error of near 8km for both Latitude and Longitude. However there are still some local areas in the Mosaic with significant errors in Longitude.

From these samples, the intrinsic accuracy (RMS) of Latitude seems to be about 8km which is about 0.072 of a degree or 4.3 minutes. This includes factors not related to the original measurements and so perhaps the RMS accuracy of the Pole Star measurements could well be about 0.036 degrees which would mean they were done very competently for the instruments available at the time and in a difficult survey environment<sup>17</sup>. The Jesuit Brothers would have had tables of the declination of the Pole Star which were essential at the time as the Pole Star was further from the true north point than it is today. But how good the tables were we do not know. Because of all the uncertainties, measured Latitude can be very variable and seems similarly variable in all areas as indicated in Figure 8.

<sup>17</sup> In 1714 Newton was attempting to devise how to use the moon to determine Longitude. His objective was to achieve measurement to 2 minutes of an arc but he could not do it. This an error of 3.33 km. His objective most likely needed a modern sextant to be realised.

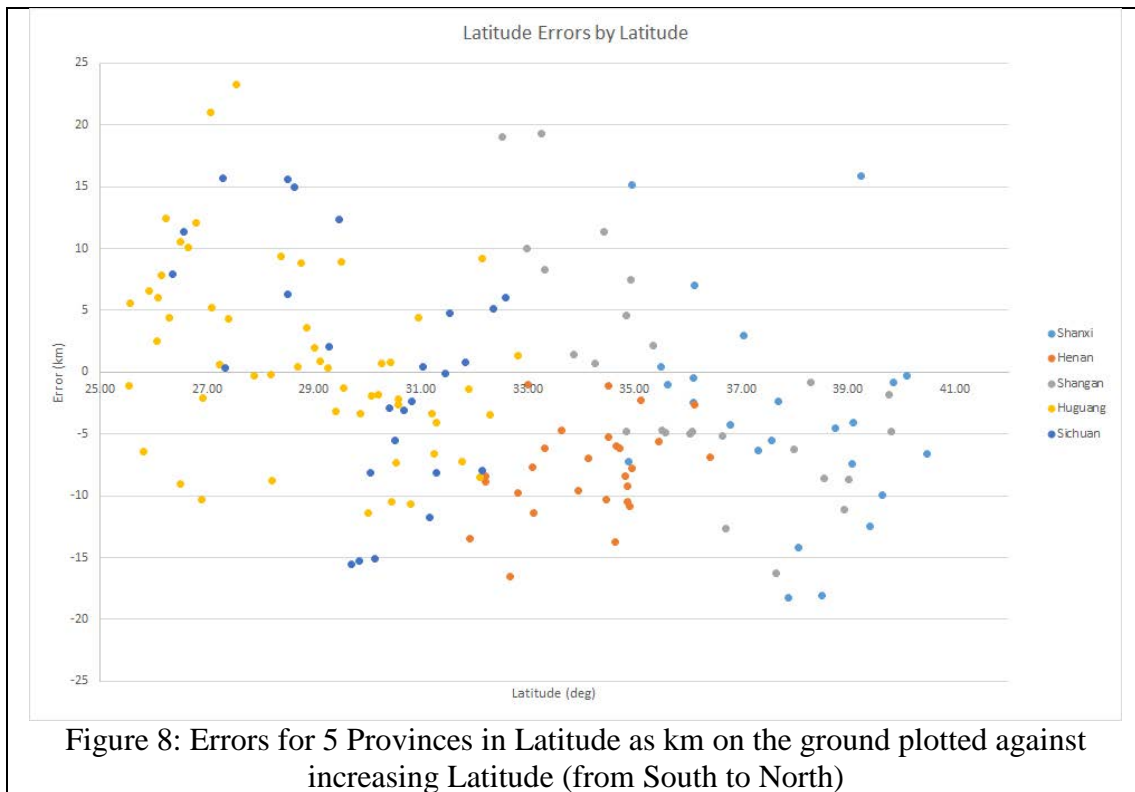


Figure 8 plots only Latitude error in km by Latitude. The scatter of Error in Figure 8 is high at all Latitudes. There seems to be a trend of Error with Latitude but in the scatter of data it would be hard to say it is significant. Perhaps it is something to do with the Pole Star estimations, but more information would be needed about the methods used to decide. It is unlikely due to an earth shape issue which could be suspected if the trend was of the order of 1-2 km but not for the variation in Figure 8<sup>18</sup>. Despite the scatter, in 1718, the present writer suspects these errors (even the -20km to +25km min to max range) would not have been unacceptably large - even in Europe. Apart from areas where surveys through mountainous terrain seems to have led to bias in both Latitude and Longitude, the main source of regional error seems to have been the difficulty of measuring accurate Longitude differences to Beijing at the Anchor Points and the distortions this creates.

### ***Location of regional errors in ShanGan and Huguang***

The two provinces where the error is most significant seem to be the combined provinces of ShanGan (Shaanxi and Gansu) and Huguang (Hubei and Hunan). It turns out that these errors are specific and regionally located.

<sup>18</sup> In du Halde [3,4] Fr. Regis reports how the Brothers noticed changes in the length of 1 degree on the meridian during a transect across level country between 41 N and 47 N. He said that at 47 N the difference “from the others” was 258 Chi (Chinese feet) or 80 metres with distance increasing as you go north. The difference in length on the WGS84 Spheroid between 41 N and 47 N would be about 59m and between 47 N and Beijing would be about 160m. The Brothers observations supported Newton’s calculations which would not have been very welcome in France at the time [23].



## Local Errors in ShanGan

If the errors in both Latitude and Longitude (in units of km) are only plotted for ShanGan province, the result is as shown in Figure 9.

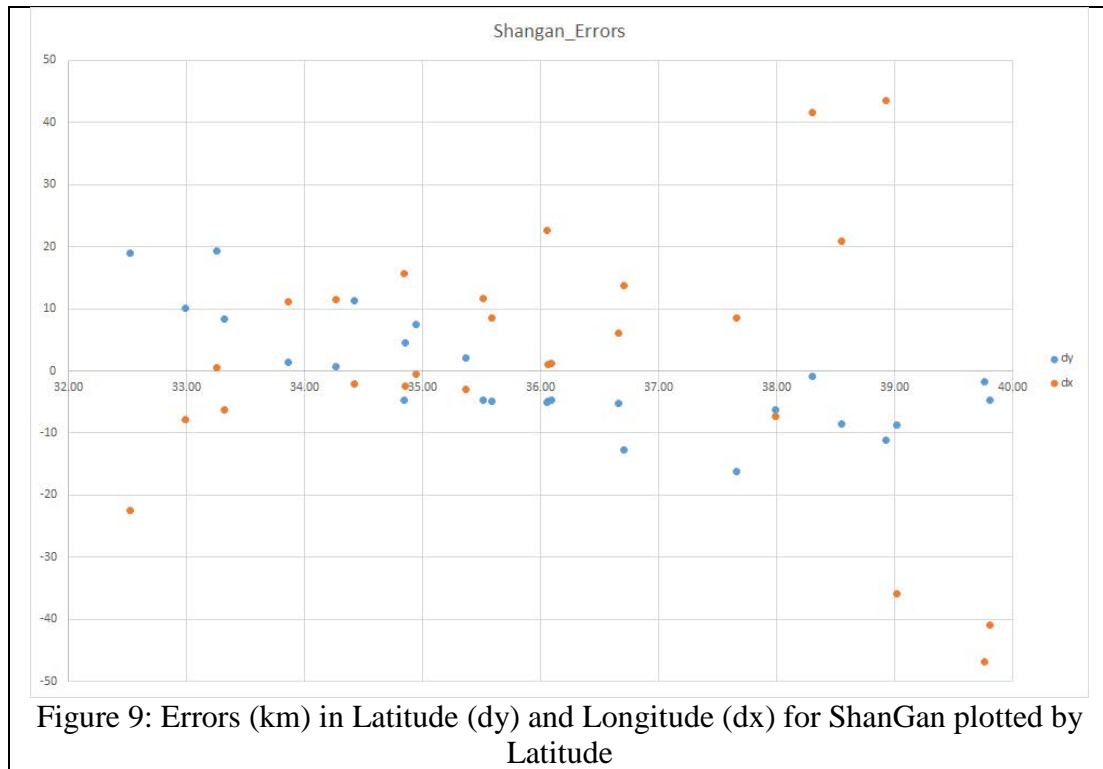


Figure 9: Errors (km) in Latitude (dy) and Longitude (dx) for ShanGan plotted by Latitude

In Figure 9, the blue dots are Latitude errors and the orange dots are Longitude errors. They are plotted against Latitude. Again there is an interesting apparently systematic trend in Latitude but whether this is real is unknown. There also seem to be two areas with large Longitude errors, one in the north (highest Latitudes) and one in the south. In the south the same places also seem to have larger Latitude errors. If the errors are located in the map it seems clear that the source of the northern problems is along the Great Wall and the southern problems occur in the Han River Valley.

The errors in Latitude and Longitude for places in ShanGan near the great wall (a limited Latitude range) were extracted and plotted against Longitude in Figure 10. The places involved, from West (left hand) to East (right hand) are Jiayu Guan (嘉峪関), Su Zhou (肅州), Gan Zhou (甘州), Xining Zhou (西寧州), Liang Zhou (涼州), Lan Zhou (蘭州), Zhong Wei (中衛), Ningxia Wei (寧夏衛), Yulin Wei (榆林衛) and Shenmu Xian (神木縣). The linear change in Longitude error (orange dots, dx) is quite striking. However, south of Lanzhou (where the Longitude error is smallest) this behaviour is not found. The Latitude errors (blue dots, dy) are similar in magnitude to other most places and show no systematic bias.

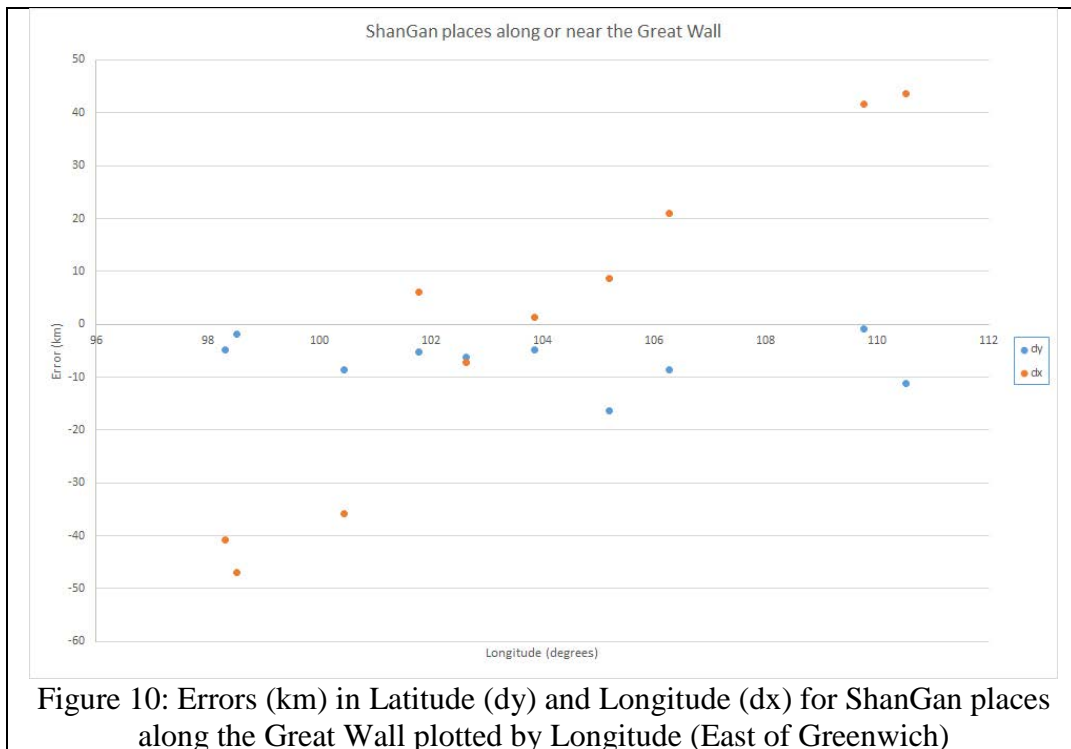


Figure 10: Errors (km) in Latitude (dy) and Longitude (dx) for ShanGan places along the Great Wall plotted by Longitude (East of Greenwich)

As to what is occurring, it is known from sources to be discussed later that Jiayu Guan and Su Zhou at the Eastern extent of the Great Wall were places where astronomical measurements were most likely originally made for Longitude. It is also possible that many places along the Great Wall were not revisited during the provincial surveys to save time, as the Great Wall had already been mapped from Jiayu Guan in the west to Shanhai Guan in the east (at the sea). Lanzhou seems to have been visited during the new survey and possibly there was a good astronomical measurement there. It is curious that these errors do not continue on for the places near the Great Wall from ShanGan into Shanxi. A full analysis will need the Great Wall to be extracted from ShanGan, Shanxi and Zhili. As this area was not a part of the mapping exercise for which the Mosaic was derived the issue will not be taken a lot further here – except to briefly discuss the screen image from Google Earth shown in Figure 11.

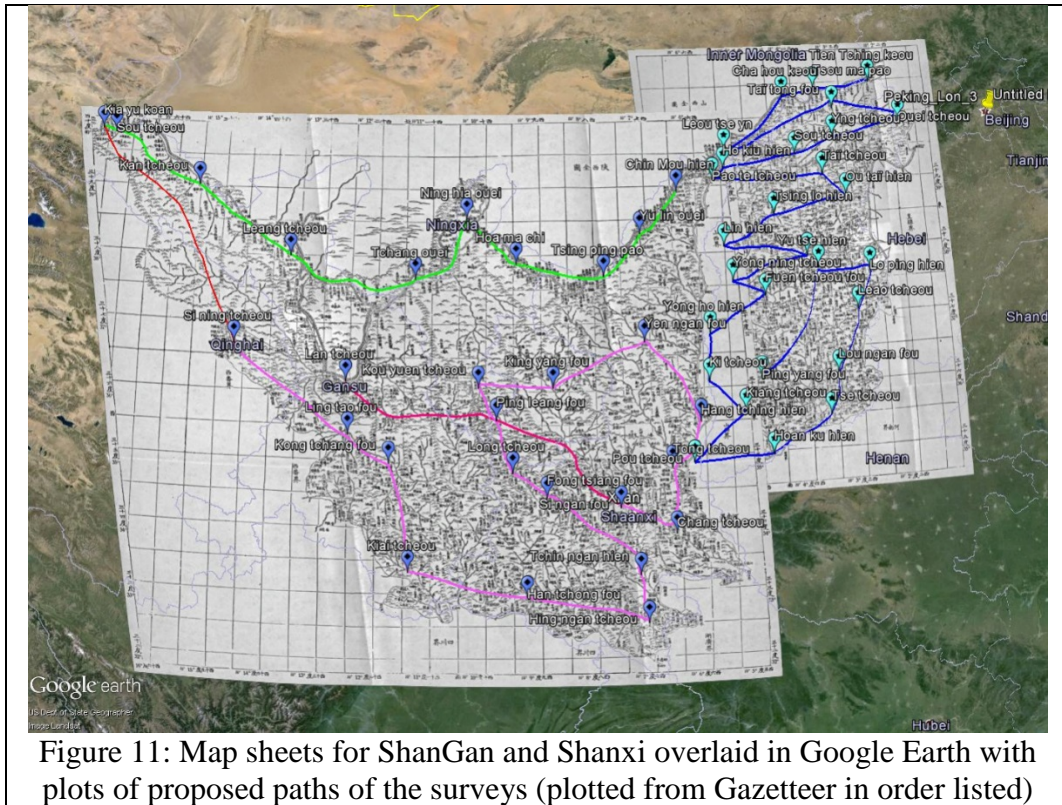


Figure 11: Map sheets for ShanGan and Shanxi overlaid in Google Earth with plots of proposed paths of the surveys (plotted from Gazetteer in order listed)

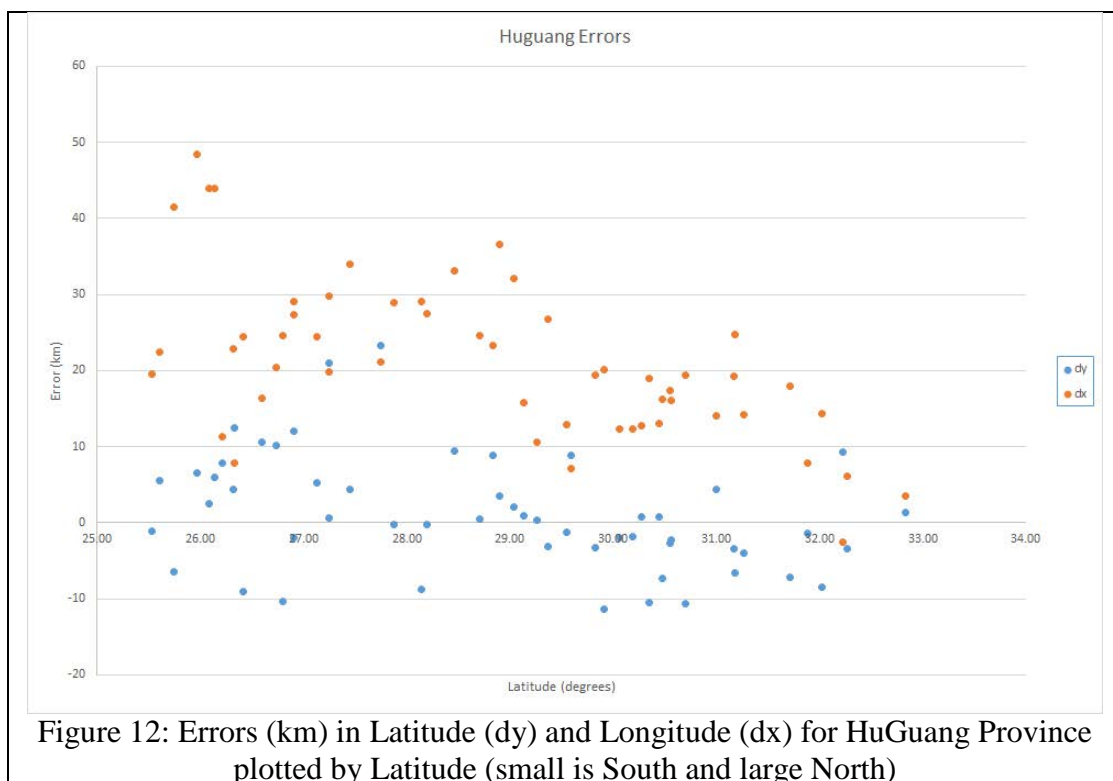
In Figure 11, the images of the maps for the two provinces of Shangan and Shanxi were re-projected to geographic projection and imported into Google Earth. The base places in the du Halde Gazetteer [3] are also plotted together with conjectured survey tracks assuming the places were visited in the order listed in the du Halde Gazetteer. This assumption does not have independent confirmation from (eg) du Halde [3,4] but when plotted it seems to make sense. It is known that Brothers Cardoso and de Tartre mapped Shanxi first and then ShanGan. Based on the list order it seems they may have started in Shanxi at Tiancheng Wei on the Great Wall (top right on Figure 11). They could have then gone in the very sensible zig-zagging route shown to end up at Pingyang Fu in the southern central part of Shanxi. In ShanGan, the list apparently starts at Shenmu in the northeast and follows the Great Wall west. From Jiayu Guan it goes south on a long but feasible track across the Qinling to the Han Valley then back across the Qinling to Xi'an in the Wei Valley. The last survey line (assuming the interpretation of list order is correct) is a single long track from Xi'an to Lanzhou along the well-established main road.

There are no conclusions in this document that depend on the order of the points in du Halde's [3] Gazetteer being the route order. However, the sensible nature of this assumption in many cases makes it worth to establish its truth or otherwise. For example, the above route implies that the team went to the Han Valley (southern Shaanxi) from the west of present day Gansu via Gongchang Fu and Jie Zhou (present day Longnan) to Hanzhong. This has not been a well-used route through Gansu in the past, so its resolution is of interest. The order based route implies that from Xing'an (present day Ankang) they followed an ancient route called the Kugu Road. This is sensible and its truth would also be of interest. The order assumption also suggests that in between the places recorded by du Halde [3] there would have been significant intermediate places with calculated Latitudes and Longitudes. If these original intermediate tracks and survey data had been preserved a very much richer and even more valuable data source would have been available.

Apart from the Great Wall section and the Han Valley (to be discussed below) the map accuracy is excellent – most likely due to very good Longitude observations at Xi’an and perhaps Lanzhou. A conjectured reason for the Great Wall issue in ShanGan is that the Brothers decided to re-use most of the Great Wall map across the north of ShanGan and by doing so introduced errors from the original survey and some inaccurate astronomical observations. The linear change in Longitude across the Great Wall is also possibly consistent with the measure of the new standard Li being in error in this section but more information would be needed to be clear about this. Suffice it to say that the east end of the Great Wall at Shanhai Guan was apparently located astronomically with great precision and that the large errors seem confined to ShanGan. The remaining larger errors are in the Han Valley and mainly at the eastern end near Xing’an Fu. These lead to distortions in the Han River (eg a shift of about 20km) which were exacerbated during map drawing by the errors that will be discussed next in the neighbouring province of Huguang.

### Local Errors in Huguang

If the errors in both Latitude and Longitude are plotted in km units for Huguang by Latitude, the result is as shown in Figure 12.



Again, in Figure 12 the Latitude errors (blue dots, dy in km) show an apparent trend with Latitude but the errors are no greater in magnitude than other places. However, there is a dramatic difference in the Longitude. For Latitudes lower than 30 degrees the errors are very high reaching to over 40km. It seems as though there is a large bias in the error but removing a constant bias (changing the location of Beijing) makes errors at Latitudes higher than 30 degrees even higher – including affecting the area covered in the mosaic. The problem is not therefore simply fixed by change in baseline. In the Mosaic developed in this document, the

areas in the north of Huguang are the most important and in this area the errors are small. But the errors in the south of Huguang are very large and a specific future study is needed to locate the specific problem with the survey.

Huguang was part of a survey that started with the Brothers Fridelli and Bonjour going to Sichuan and then Yunnan. Owing to the death of Fr. Bonjour and illness of Fr. Fridelli, Fr. Regis went to Yunnan and, with a recovered Fr. Fridelli, completed Yunnan and moved on through Guizhou and Huguang. A more complete study must therefore include all of these provinces and consider the survey sequence and route. The Longitude accuracy of Sichuan is best of all Provinces studied here so it seems possible the change in baseline may have been associated with the second phase of the survey after Fr. Regis arrived. In the next section it is shown that the anchor point at Guangzhou is possibly the worst there is in error among all likely candidates. If Fr. Regis tried to make use of this single anchor point it could have seriously biased the survey and errors would likely have been compounded in the mountainous areas of Guizhou and Huguang. The baseline errors in Huguang caused the Changjiang River to be significantly regionally shifted (of the order of 10km in the north and 20km in the south) away from its true location and merging the information from ShanGan and Huguang maps in the map drawing process most likely affected the upper (but not the lower) Han River.

### ***River Valley errors in the Mosaic area***

One objective of this work was to obtain a map that could be used as base for studying the extensive traditional Chinese mapping by Yan Ruyi and others between 1813 and 1822 in the frontiers of Shaanxi, Gansu, Henan, Sichuan and Hubei covering most of the Han River Valley catchment area. The mosaic produced above is relatively free of major errors such as those found along the Great Wall but is affected by the Longitude errors in the north west of Huguang (which are less than in the south) and some errors seen in the Han Valley where the Han River flows into Hubei Province. A set of interesting and localised places in this region can also be checked and have been summarised in a set of sub-Tables all with “Table 7” as the main heading:

Confining the evaluation to the 21 du Halde mapping points within the mosaic area we obtain the following overall Mosaic summary (converted to km as before using the relationships between angles and distance):

Table 7.1: All Mosaic summary Errors

Mosaic Summary	Latitude (km)	Longitude (km)
Av (km)	-1.0156	4.9480
Stdev (km)	8.2225	8.9680
RMS (km)	8.2849	10.2424

This is better than the overall average of all mapping points over the five provinces and much better than either ShanGan or Huguang alone. For reference, the all-province averages (in the same format as above) were:

Table 7.2: All Provinces summary Errors



All Points	Latitude (km)	Longitude (km)
Av (km)	-2.1664	6.0923
Stdev (km)	7.6729	10.9483
RMS (km)	7.9729	12.5292

To make a more targeted local assessment of the mosaic errors, it was decided to look at the primary river valleys and assess how accurate the mapping is in these places. The Mosaic can be mapped into Google Earth as seen in Figure 13. The river valleys used are sections of the Wei River (that runs across west to east to the north of the Mosaic); the Yellow River where it comes down almost due south as the border between Shaanxi and Shanxi to meet the Wei River and turn East at the Northeast corner of the Mosaic; the (Upper) Han River (that runs almost horizontally across the middle of the Mosaic from West to East); the Jialing River (that runs north to south on the western side of the Mosaic) and the Yangtze River where it cuts as a curved arc across the south east of the Mosaic.

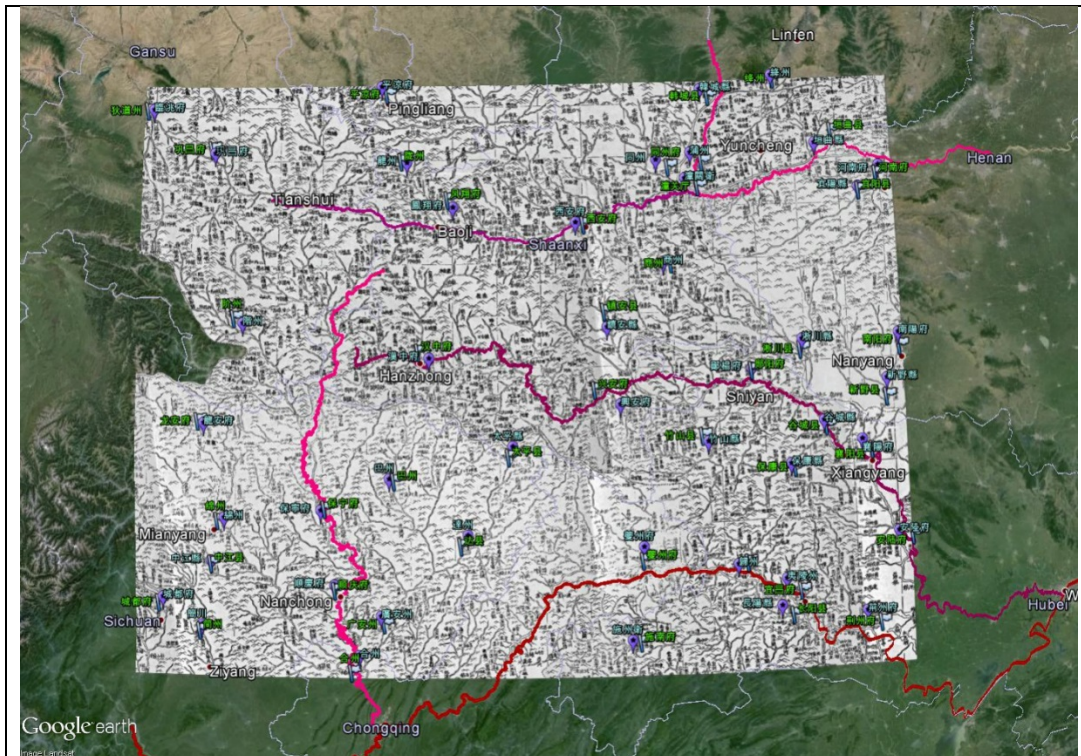


Figure 13: The Mosaic area with plots of the five river valleys used to test accuracy (image presented in Google Earth as super-overlay)

The way the evaluation has been done is to locate places along the rivers in the map, read the projection coordinates and match them with the same place in the ChinaW set. There are enough places here to be useful although other places could also be used if needed. The overall summaries for the five river valley cases are as follows:

### Han River Valley

Table 7.3: Han River Valley summary Errors

Han_River	Latitude (km)	Longitude (km)
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Av (km)	5.2643	-4.5544
Stdev (km)	8.6163	13.1244
RMS (km)	10.0972	13.8921

There are two sources of error in the Han River valley. One seems to have come in the survey with bias in both Latitude and Longitude that is worst (perhaps due to a poor astronomical reading) at Xing'an (present day Ankang) at which place the Han River is about 20km from its true position. Things only come together when the upper Han becomes the Lower Han (just upstream from Jun Zhou). It is possible that the divergence of the Han River from true position in between is a drawing issue as the rivers need to be joined across the Province boundaries when the map is drawn. The surveys in Shaanxi and Huguang were done by different people at different times and there is an area near the common border where no ground survey data was taken at all.

## Jialing River

Table 7.4: Jialing River summary Errors

Jialing_River	Latitude (km)	Longitude (km)
Av (km)	-1.8084	0.3134
Stdev (km)	8.8535	5.2410
RMS (km)	9.0363	5.2504

Apart from some places near the Jiang River where because of the Huguang issue some Latitudes were about 15km to 19km off position, the Jialing places were all closely placed to true position as indicated by the overall low RMS values.

## Jiang River

Table 7.5: Chang Jiang River summary Errors

Jiang_River	Latitude (km)	Longitude (km)
Av (km)	-9.2636	8.5278
Stdev (km)	5.2380	10.8288
RMS (km)	10.6419	13.7835

The Jiang River (Yangtze or Chang Jiang) in Huguang has a large bias resulting from the Huguang problem discussed earlier. It seems to be due to one or more poor anchor points. Longitude is the main problem with the river drifting over an area for up to 10km in Latitude and 16km in Longitude at its greatest divergence.

## Wei River

Table 7.6: Wei River Valley summary Errors

Wei_River	Latitude (km)	Longitude (km)
Av (km)	0.6965	2.6092
Stdev (km)	3.5125	6.5601



RMS (km)	3.5809	7.0599
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Along the Wei River is one of the most accurate runs of data seen in these five provinces. It seems that Xi'an had well established astronomical readings – possibly taken over some years with RMS Latitude being about 3km. The statistics are only spoiled by three places near the bend in the Yellow River at the East end of the valley. These three are affected by the issues in the Great Wall as are the next set.

## Yellow River

Table 7.7: Yellow River Bend summary Errors

Yellow_River	Latitude (km)	Longitude (km)
Av (km)	-3.4804	12.9838
Stdev (km)	5.2978	4.9188
RMS (km)	6.3387	13.8843

The Yellow River data are shifted significantly with 13km bias in the Longitude. It seems to have been inherited from the errors found in the Great Wall data in Shaanxi. These were discussed earlier. If that bias were not present the data would be excellent.

Finally, the total summary statistics for all of the data used in these river valley areas is presented below:

Table 7.8: All River Valley summary Errors

Total	Latitude (km)	Longitude (km)
Av (km)	-0.8348	3.1192
Stdev (km)	7.9399	10.5536
RMS (km)	7.9837	11.0050

Values of 8km and 11km for RMS are as good as is generally found in the Kangxi maps except (as always) for some of the local errors that are well out of range – especially in ShanGan and Huguang. However, by leaving out the areas where specific and significant distortion errors occur (on Jiang, Yellow and Han Rivers) an estimate for what may be possible if a better network of astronomical observations were in place can be developed from these data as follows:

Table 7.9: Summary Errors with high distortion areas removed

Best	Latitude (km)	Longitude (km)
Av (km)	-0.7230	0.4799
Stdev (km)	6.5466	5.5941
RMS (km)	6.5864	5.6146

RMS values close to 6km corresponds to an error in estimating the Latitude (for example) by measuring the pole star of 3.6 minutes with the Wei River Latitude RMS being nearly one half of this (<2 minutes). If the whole map had this level of accuracy it would very likely be better than any European maps of the time – as already claimed generously by Joseph Needham [17] for the whole collection. Nevertheless, the Mosaic is sufficiently accurate for

the purposes of this study to help collate and scale the maps prepared in this region by Yan Ruyi 100 years later. As an example, scaling one of the main Chinese gridded maps that arose from the work of Yan Ruyi to a rectangular grid using control points achieved a 15km RMS error with some of the points being up to 30km from true location. The Kangxi maps are generally more accurate than can apparently be achieved by simple scaling of Traditional maps – but not an order of magnitude different. The difference is the globally valid projection framework.

## Errors in Anchor Points

The errors found are likely to be build-up of survey errors over long distances away from places where “anchor” points, or careful astronomical observations, were located or in observational errors at the anchor points. The present writer does not confidently know which places are referred to by Fr. Regis as quoted by du Halde [3,4] as being where “*observations of the Moon and satellites of Jupiter made before our time by members of our Society*”. It is known, however, that the data listed by Souciet, [18] were astronomical observations and made by the Jesuits. If those listed by Souciet, observed before 1721 and located within the area covered by the maps are identified and the subset that is also listed in du Halde’s [3] Gazetteer are selected the number is smaller. Finding the equivalent ChinaW places allows estimates to be made of accuracy at these possible anchor points. Taken overall, the mean and standard deviation of accuracy of Latitude and Longitude for this selection of places came out as listed in Table 5.

Table 5: Overall accuracy (km) of proposed anchor points

	Latitude (km)	Longitude (km)
Av=	-4.720	-2.119
Stdev=	4.519	19.849

The bias values could just be due to sampling. It seems that Latitude Standard deviation is much less than found for the survey places in the maps. The Longitude standard deviation, on the other hand, is high. The details of the points and error summaries have been tabulated and are presented here in Appendix 2. There are three places where the Longitude errors are very high, being close to 40km in each case. If these three are not included the standard deviation for Longitude is 11.39km which is better than those found in the du Halde base control points. But it is still more than twice the Latitude value in Table 5. The biggest individual errors among the three (about 41km and 45km) are at the western end of the Great Wall at the close places of Jiayu Guan and Su Zhou. These were also the cause of the main distortion discussed for ShanGan. However, in the north there are many other accurate anchor points and also survey lines that go from Beijing to the place in error as cross checks. The errors are therefore usually confined to near the erroneous points. In the case of Jiayu Guan and Su Zhou the error propagates along the Great Wall but is confined to ShanGan. The third big error among the selected anchor points is at Guangzhou in the south where the Longitude error is close to 40km. There are no other places in the list of potential anchor points from the south east and it is therefore possible that Guangzhou was a key (if not the only) anchor point in this area.

It is also possible to investigate how important the error in the Guangzhou anchor point may have been by considering the itinerary of the Jesuit Brothers. As stated above, after mapping

ShanGan and Shanxi, Brothers Cardoso and de Tartre went south to map the Provinces of Jiangxi, Guangdong and Guangxi. It is not known if there was any link between surveys near Beijing and the starting point in Jiangxi. But Guangzhou was certainly a possible anchor point for the southern survey. In regard to the provinces involved in the mosaic, as was quoted from du Halde [4] above, “*Fathers Fridelli and Bonjour were sent to map the provinces of Sichuan and Yunan. However, Father Bonjour died near the border with Burma and Father Fridelli also fell ill. Father Regis was later sent to complete the map. Father Fridelli had recovered and together they went on to map Guizhou and Huguang*”. The survey entered Sichuan from Shaanxi with only a small Longitude error and the survey in Sichuan seems to have maintained high accuracy. The anchor points in Sichuan are also generally useful and accurate. Presumably, the initial areas surveyed in Yunnan were similar. But when Fr. Regis arrived he may have come in from the sea and there may have been no link with the previous surveys to start from. If a survey line from Guangzhou provided the initial reference for Longitude, the Guangzhou error could have been propagated through parts of Yunnan, Guizhou and Huguang where no anchor points seem to be located. This could explain the distortions seen in Huguang but more work and more detailed information is needed to resolve it further.

It is also likely that the places previously measured astronomically would have resolved Longitude as East of Paris (or in some cases Marseilles). Changing to Longitude west or east of Beijing involved additional error. As Fr. Regis says in his discussion quoted by du Halde [3,4], the survey method used by the Brothers was usually much more accurate than astronomical measurements. Unfortunately, independent and absolute anchor points are needed to fully and accurately locate the maps, especially with such large distances being involved. As a consequence it may be asked what would happen if improved anchor points and additional accurate anchor points became available? If the original data were still available the positions of all the base points could be re-calculated with the revised anchor points and the maps redrawn – but this does not seem possible now.

## ***Comparison with the Martini Maps***

The Kangxi Maps and the derivative set of Jesuit maps published in Europe in Jean-Baptiste Bourguignon d'Anville's new Atlas [1] were not the first such works. Seventy or so years earlier, as Ming gave way to Qing, the fortunes of the Jesuit Brothers in China were variable until the Kangxi Emperor came to the throne. Between 1625 and 1665, the Jesuit Brother Martino Martini (1614-1661, Wèi Kuāngguó, 卫匡国) returned to Europe bearing the fruits of Jesuit geographic and cartographic studies of China to that time. He published his set of maps of China by Province in 1655 through the offices of the famous European cartographer Johannes Bleau. Martini's maps and associated descriptions of China's geography were collected in a book written in Latin called “Novus Atlas Sinensis” or “The New Atlas of China” [20]. It became the base for knowledge of China in Europe until the maps being analysed here were developed and sent to Europe. A combined map of the individual Province maps is shown in Figure 14 and shows an overall similarity of scope to the Kangxi maps as summarised in Figure 1.



Figure 14: Combined map of the Martini 1655 China Province Maps

The earlier maps were less polished but were quite similar in presentation. The individual Province maps are also on sinusoidal projections with Beijing as reference Longitude<sup>19</sup> and present places down to District level and also many forts and large townships. It seems the development was similar to the later maps in that places would have been located on the map with pre-drawn projection grid and information scaled to the reference locations from Chinese maps. The difference is that Martini and the Jesuit Brothers of his day seem to have only had Chinese maps and existing Gazetteer route distances in Li to estimate airline distances and then convert to latitude and longitude differences using an estimate for metric scale to 1 degree. Martini used 250 (short) Li to one degree change in Longitude and certainly knew to scale one degree change in Longitude by the cosine of the Latitude. No doubt the map suffered from the variable definition of the Li throughout China but at the time it was a reasonable approach to take. At the earlier time, the declination tables for Pole Star and sun position estimates of Latitude were not available or not accurate and accurate astronomical estimates of Longitude were not yet as fully developed as they were for the Kangxi maps.

In effect, the Kangxi maps were what the Jesuit Brothers had really wished for in the Martini Maps but which had only become established in France in the late 17<sup>th</sup> century [23]. For the later mapping, data were based on locally chained ground survey as well as astronomical observations and the metric for the degree was accurately calibrated to a standard. But although the Martini maps do not seem to have the standard of the later maps, it is useful here to establish more than a visual idea of how improved the Kangxi maps were relative to Martino Martini's maps of China. For indeed, nothing like the Martini maps had existed

<sup>19</sup> The combined map in Figure 14 is apparently on a conical projection like Figure 1. Longitudes are given both from Beijing and from the Azores, an older reference point for Longitude.

before he and Athanacius Kirchner [19] gave Europe the first Jesuit geographic data base for China and displaced Marco Polo and maps inspired by his travels as the primary reference.

As with the Kangxi Maps, “Novus Atlas Sinensis” [20] included a Gazetteer of places in China with estimates for Latitudes and Longitudes relative to zero at the Beijing Meridian. The coordinates are only listed to the nearest minute. By including only places to Fu level (Prefectures and Provinces) it is found that there was not much change between the maps in names of places. In this way we have coordinates for the same places in the Martini, du Halde and ChinaW systems. So it is possible to estimate accuracies between Martini and ChinaW and du Halde and ChinaW. The differences between the Martini and du Halde places and the corresponding ChinaW places can be measured in kilometres when scaled by the Latitude and Longitude distances in km per degree. The complete set of places and errors involved are summarised as Tables in the Appendices 3 and 4. Based on these Tables, the summary statistics of the differences are as listed in Table 6.

Table 6: Comparative overall five Province accuracy for du Halde and Martini

Error (km)	du Halde		Martini	
	y_err	x_err	y_err	x_err
Av=	1.3474	-8.8171	83.845	-87.893
Stdev=	8.1057	12.6600	55.932	46.660
RMS=	8.2170	15.4278	100.789	99.511

In Table 6, the column headed “y\_err” is the error in “y” or north-south (i.e. in Latitude) and the column headed “x\_err” is the error in “x” or east-west (i.e. in Longitude). The three rows are the average error (to locate bias), the standard deviation about the mean (to assess variance) and the RMS or Total Error including bias and variance. For the du Halde places in the set, The RMS is about 8km for Latitude error and 15km for Longitude. There is an 8km Longitude bias as well. These are similar to the levels found previously for all of the du Halde points in the five provinces. The RMS for the Martini map is about 100km in each coordinate. It is clear there are large positive bias errors (84km and 88km) in the Martini maps but even if bias could be removed, the standard deviations are still 56km and 47km. These are very large errors by modern standards. But at the time, the maps were a revolution and set the stage for the more methodical survey and mapping at the beginning of the 18<sup>th</sup> Century.



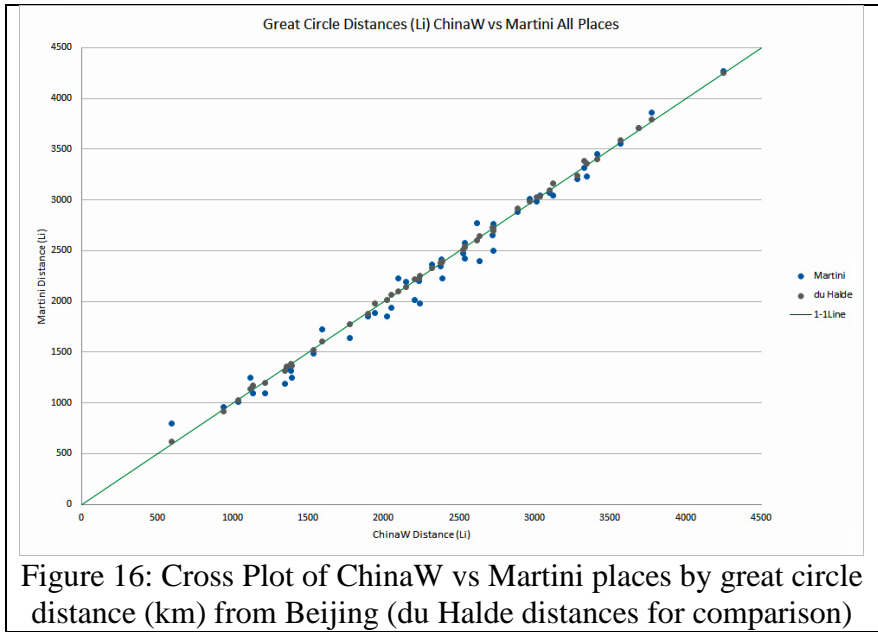


Figure 15: Plot of points in the five Provinces from the Martini Gazetteer and ChinaW set with lines joining same place.

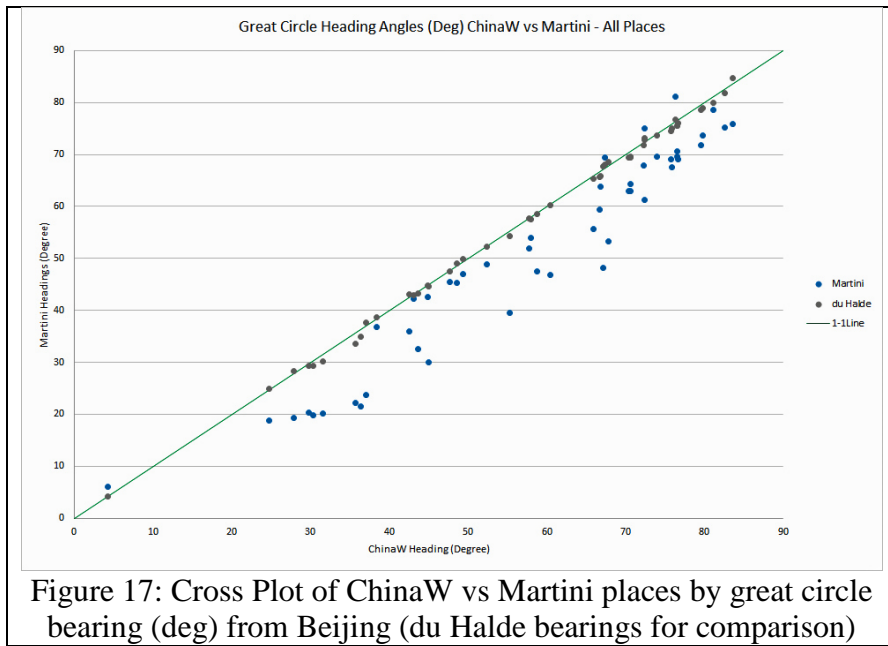
It can be asked whether there is some specific cause for these errors. Figure 15 plots the Martini and ChinaW points in Google Earth and also shows lines joining the two estimates of location for places. Almost all errors represented by the orange lines are a shift from South East to North West and they can be very large – especially in the north of China. This fact identifies the bias. They also seem to lie along arcs of approximately “concentric circles” around Beijing.

A possible conjecture is that Martini used distances and bearings to estimate the coordinates away from Beijing. Hence, some insight into the behaviour illustrated in the Google Earth screenshot above can be obtained from plotting the data sets in terms of distance and bearing from Beijing. Every one of the places in Figure 15 has Latitude and Longitude coordinates in the Gazetteers provided by Martini [20] and du Halde [3] and is also able to be found among the accurate ChinaW set [16]. For any set it is therefore possible to find the distance between Beijing and each other place along the great circle (or geodesic) distance and also calculate the heading angle for the direction of the place from Beijing. Martini [20] also includes a Table of distances between Provincial Capitals including Beijing. Compared with entries in that Table, only one of the five provinces used here had significant difference in recorded distance compared with the estimated geodesic distances and that was Chengdu. This is despite Chengdu coordinates being one of the more accurate sets of coordinates. It suggests some transcription or other similar error.

Presenting the data as graphs rather than in Tables, the first investigation is to plot the ChinaW great circle distances (X) against the Martini great circle distances (Y) in km. The same result for the du Halde places is also plotted for comparison as is the 1-1 Line (perfectly aligned data case) in Figure 16.



The Martini distances (blue dots) show some scatter but relative to the RMS errors in Table 6 the scatter is very small. The grey dots are the plotting places for the Kangxi maps as listed in du Halde and show useful improvement over the earlier mapping. So in this aspect, the Martini maps seem very good for maps made at the time. However, the story is different for the heading or bearing angles as illustrated in Figure 17.



For the bearing angles in Figure 17, there are some very significant departures from full agreement and relative to the Kangxi maps as well. These are the lines roughly on arcs of concentric rings around Beijing seen in Figure 15. They are almost all displaced to the North West for the 5 provinces being used. They certainly seem to be due to systematic errors in estimated bearing for the great circles.

It is not clear how Martini calculated the coordinates. Perhaps he used distance from Beijing and an estimated heading for province capitals and then used the “triangle” calculation for



Latitude and Longitude of the capitals. He may have then spread estimates out from the province capitals in a similar way to the later survey (using the triangle method) but without the ground measurements – only map based estimates. Distances may also have been estimated using a “straightened” route distance from gazetteers such as would have been developed for the Ming postal road system which had certainly been well surveyed. For details and possibly headings he would probably have used Chinese maps.

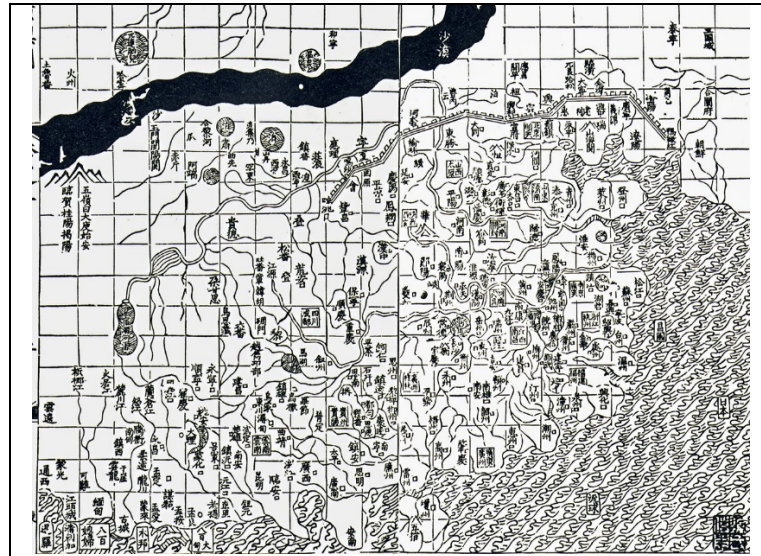


Figure 18: Map originally by Zhu Siben (輿地總圖) possibly similar in geometry to one known to Martini.

What is seen above in Figures 15-17 may have arisen from the typical distortion of ancient maps of China into a “square” shape which occurred in almost all traditional maps. For example, in Volume 3 of “Science and Civilisation in China” [17], Joseph Needham and Ling Wang wrote (22 (g), p.586) “it is well known that Fr. Martin Martini’s *Atlas Sinensis* of +1655 was largely based on the *Guangyu Tu* (輿地總圖), which European geographers such as d’Anville greatly admired”. A later copy of this map was included by Needham and Wang [17] and in somewhat modified form (as a mosaic) it is shown here as Figure 18. This map uses the method of squares and comes from an atlas compiled by Luo Hongxian (羅洪先, 1504-1564). It was apparently re-scaled from another famous map by Zhu Siben (朱思本) whose original map had been produced between 1311 and 1320 [17]. Needham and Wang quote Luo as writing that Zhu Siben’s map (the one used by Luo Hongxian) was 7 feet long. If Fr. Martino Martini had used a map such as this (or used the original larger scale map) for directions and used published postal route distances perhaps it explains what is being seen in the above plots and Figure 15. It is certainly clear that although the Martini maps were a revolution in Europe in the mid-1600’s, the updated maps of the Jesuit Brothers who surveyed the Kangxi maps from 1707-1718 were an order of magnitude improvement in intrinsic map accuracy.

## 5. Discussions

### ***The problem of Longitude***

The main limitations to the accuracy of the Kangxi maps were from the astronomical measurements of Longitude. Fr. Regis was right to say that for places close together the survey was more accurate than using the astronomical measurements. Provided the distances between base mapping places for which the survey was done were not more than 2 or 3 degrees of Latitude or Longitude the geometry could be treated as Euclidean and the estimated differences in Latitude and Longitude will be accurate – unless the terrain is very fierce. But the error would build up continually if this was done to far places – perhaps like those in the Martini Table. The whole map therefore has to be fixed by a good set of anchor points where the Latitude and Longitude are accurately known. In the case of Longitude, they were not always available for the Kangxi map. One way an accurate map base for China could have developed is for a set of observatories to have been established at major cities across the country and for a number of accurately fixed baseline transects to be surveyed and maintained. The observatories could have developed Ephemerides and observations of the sun and stars<sup>20</sup> over a number of years. Established transects could have provided reference points for revisions as well as the additional traverses and the routes that spread out from them. But such a network had to be set up and supported over an extended period by the Emperor and his successors as was occurring in France at this time [23]. In China, as the Kangxi maps were mainly a product for that one Emperor whose reign ended in 1722 and for various reasons, including the Chinese Rites Controversy, later Emperors had less interest and capacity than the Kangxi Emperor - it never happened.

But there was also another problem and it was more fundamental. Since ancient times, Longitude had been very hard to estimate by any practical method and was generally roughly established by converting between distance, bearing and spherical angles. The problem was most critical at sea where currents and winds were difficult to guess and errors of 100's of km were quite easy to make. In the period 1707-1719 the most advanced European methods were astronomical as practised by the Jesuit Brothers. However, 100 years later Longitude on the sea and the land was increasingly being determined by robust clocks – or chronometers [13,14]. The reason is that the measure of Longitude is time. Local time (or Local Apparent Time, LAT) is based on midday being when the sun is at its highest in the sky. A pendulum clock can be set at that time to record the LAT at other times. The difference in LAT between two places at the same instant is precisely related to Longitude difference. But it was hard to move clocks and have them still keeping accurate local time at the reference location – especially at sea. An alternative was for people to observe the same astronomical event at different places and record the times at two places for the same event in LAT. Later they can be compared. But as Fr. Regis noted:

*“The Observations of the Satellites require, not only more Time and Accuracy, but also Telescopes of the same Size, and if I may so speak, the same Eyes in the Observer and his Correspondent; for, if one sees them ever so little sooner than the other, some Error will inevitably happen.”*

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<sup>20</sup> Souciet [18] includes records of observations of the crossings of the moons of Jupiter in Peking carried out between 1722 and 1726. It would be interesting to find out if improvements were made to anchor points in the later Qianlong updates of the maps.

An alternative, that was becoming standard, was for regular astronomical events to be predicted and the times of observation at (say) Paris into the future calculated into Tables. Then only a single observer is needed but at that time Tables were not always accurate and the recorded data listed by Souciet [18] show wide variations between observations and alternative Tables (which he lists). Even if the measurement is made carefully the Longitude difference to Paris would not have been very accurate when shifted to a Longitude difference from Beijing. In Europe at the time there were many observatories but even so it was not until Chronometers developed much further that accurate and easily obtained Longitude became a reality [13,14]. Even when, in the 19<sup>th</sup> Century after the Opium Wars, western traders and missionaries moved into China, very few took astronomical measurements. Latitude was occasionally taken (with less accuracy than the Jesuit Brothers!) and increasingly altitude was measured but Longitude was still for the specialist surveyor. Perhaps complaining about 20km errors in a map of anywhere in the world that was printed in 1718 is more than a little ungenerous!

Finally, while the Latitude errors were generally contained and smaller than Longitude, measuring Latitude accurately was also not a simple matter. Measurement of the altitude of the Pole Star is not enough as the Pole Star is not quite on the north point. Accurate measurements need some multiple observations with reference stars and Tables of declination. The Jesuit Brothers had the best Tables Europe had produced at the time and it was necessary as at that time the Pole Star was significantly more distant from the true north point than it is today<sup>21</sup>. Because the survey teams were continually on the move, establishing local time and making observations when weather can be bad and visibility variable was a great challenge. We can only wonder at their achievements in such situations!

### ***The impact of the Kangxi maps on Chinese mapping***

The accuracy seen in the five provinces analysed could well be much better in the Provinces generally along the coast from the Bei Zhili to Fujian. Although in the south east there is some Longitude bias for reasons discussed above, overall even what we have considered here is a very accurate set of maps of anywhere in the same time period and at the scale of presentation. The quality of the work must have impressed the Russian negotiators. However, the techniques introduced and the maps produced do not seem to have replaced traditional Chinese mapping in China until after the fall of the Qing. One reason was possibly that the court regarded the maps as their own resource and largely kept them “top secret” even as work continued with new editions appearing in the Qianlong period and in the 19<sup>th</sup> Century. But more likely there were other factors at work as well.

The impact of these maps on Chinese mapping and (indeed) the nature of Chinese maps has been the subject of much discussion. Joseph Needham and his collaborators [17], for example, believed Chinese played a large part in the mapping and the activity represented an exchange of knowledge between east and west. They also support the historical development of Chinese maps as becoming more cartographically accurate with time and (finally) with the Kangxi Maps developed to include the astronomical ideas of Latitude and Longitude and a spherical earth into Geographical Maps. Prof. Cordell Yee [5] has disagreed with these

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<sup>21</sup> In Souciet [18] the declination of the Pole Star was measured by the Jesuit Brothers on the Great Wall in 1708 as 3 degrees and 5 minutes. Today it is only 40 minutes and 52 seconds from the pole.

suggestions and proposed that Chinese maps were always better interpreted as artistic and illustrative (often literary) text rather than being cartographically scaled or metrically accurate. He believes the Kangxi maps and their later updated versions were never “Chinese” but simply western maps provided to the Qing Emperors by (contracted) foreigners. He also does not support that there was a tradition of survey and mapping in China that was cartographic and valued scale and accuracy and produced maps for practical purposes rather than artistic expression. On the other hand, the Scientific Humanism of Needham and his associates [17] would naturally support the existence of such a tradition in addition (if not in parallel) to the illustrative and imaginative maps that make up the majority of the maps that still exist in collections today.

It is hardly the place to enter such a debate except to note that the work reported here takes a somewhat central position. It is clear that traditional Chinese mapping did not absorb all of the western ideas outlined in this document. The maps served the needs of the Qing rulers but seemed not to impress the writers of Gazetteers – even if they knew of them. On the other hand, Chinese played a large part in the field survey and the techniques – including measuring the Pole Star, were already known to Chinese who took part. The basic field survey technique, as long as it did not extend more than 2-3 arc degrees, was already known by Pei Xiu and implemented by his adherents since. Judging from Han Qi’s investigations [11] it is also clear that Chinese mathematics and astronomy were positively influenced by the interactions with the French mathematicians who carried out the surveys. However, what was certainly new was the use of a spherical earth, Latitudes and Longitudes for places, scaled map projections for the maps and astronomical measurements of Longitude. Finally, Chinese who took part certainly seem to fit the description of artisan surveyors. But this is best returned to in later work when the corresponding traditional maps are also considered.

### ***The persistence of traditional mapping methods in China***

It is also possible to ask why the accurate western maps did not displace traditional Chinese mapping and traditional maps among the practical people mapping at local (Gazetteer) scale. There are some cartographic issues that may explain it. One is that traditional Chinese mapping (as does modern topographic mapping) prefers to work on a square grid system where Euclidean geometry is used. This can be done with maps as long as the extent of the map is not more than about 2-3 degrees. The grid system used since ancient times makes it possible to tell “what is near and what is far” and find estimates for overall distance “as the bird flies”. But the Jesuit maps did not present information in this form. Needham [17] and Yee [5] both mention that when Chinese re-circulated the Kangxi Maps in the 19<sup>th</sup> Century, they often drew both a traditional grid and Latitude and Longitude lines on the same map. They also produced copies that had no lines at all. If any of the survey officials engaged in traditional mapping had seen them it is likely they were not impressed, even with the gridded versions. The sinusoidal projection was ideal for sending information back to Europe. The whole map could be divided into rectangular sheets and re-assembled later and the Latitude and Longitude quadrilaterals could be used at any time to re-project and re-scale the map to other projections. It has a nice property of being an equal area projection but the local distance and angle distortions in this projection can be very large. So it is possibly the worst form of the map for Chinese to try to use to draw a square grid! The maps can, however, easily be re-projected to a suitable projection as to make them suitable to match a Chinese gridded map. This is planned to be done using the Kangxi maps in a separate study of local Chinese gazetteer maps. A standard Chinese system of map sheets that are gridded at more

detailed scales and geographic when very large could certainly have been defined at the time – although as noted by Needham and Wang [17] this kind of map series only became common in Europe in the 20<sup>th</sup> Century.

Another possibility is that the Kangxi maps were not necessary or even very useful for the purposes of local Gazetteer developers. It seems possible that Chinese maps made for practical purposes (military, road and water engineering, travel, business etc) were primarily topologically accurate but rarely metrically accurate or consistent in the local scale. This allowed local scale to suit other purposes. Sometimes overall scale is indicated by the grid over the map to indicate approximate distance from north to south and east to west (to give the user a sense of “what is near and what is far”) but to find travel distances between places the traveller would not have used the map. Rather, route distances were available in the primary Gazetteer and the accompanying map was only used to assess occurrences of rivers and mountains or place orders in regard to the route. Finding which place was next on the route; which side of a river the-road occurred and when the route was through high mountains or dense forests were likely the most common uses of the map<sup>22</sup>. Bearing and distance “as the bird flies” are generally of little use to travellers on the hard and winding roads in much of old China.

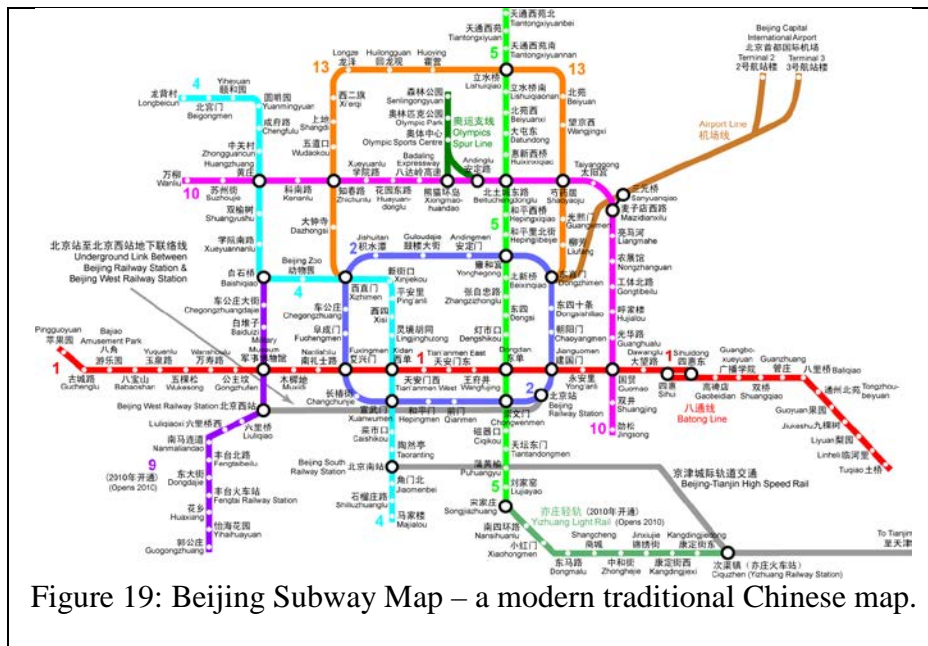


Figure 19: Beijing Subway Map – a modern traditional Chinese map.

When local scale is not critical, the rivers and mountains can be arranged to suit map annotation and appearance (such as “moving eye” scale and filling the page in the traditional “square” shape) without sacrificing topological correctness. In this way, traditional Chinese maps are often more like “subway” maps (see Figure 19) than modern survey maps. In this situation, judging how “accurate” an ancient Chinese map is may need a different measure from that which we have applied here to the Kangxi Maps or those used for modern metric maps. As emphasised forcefully by Cordell Yee [5], the maps were always (often artistic) adjuncts to more detailed text but were nevertheless important components of the intended communication. Most likely correctness in topology and connections will provide the best way to judge them. These questions have been discussed in a similar context by Brian Lees in

<sup>22</sup> Travel distances are often collected by Division such as Fu or Xian and finding the “next” place in an adjacent Xian (where the lists restart from a new centre) or catchment can sometimes be a practical use for a map.

[21] and also in [22] under the heading “Spatial thinking”. However, it is beyond the scope of the present document to pursue these ideas here.

## 6. Conclusions

The Kangxi maps were an outstanding application of the emerging European land survey and mapping techniques combined with traditional Chinese surveying to map a large area of East Asia. This resulted primarily in the maps of being incorporated into western Atlases with accuracies commensurate with European maps of the time. They did not have a great impact on the practice of traditional Chinese mapping but probably instigated developments in Chinese mathematics and astronomy. These would later come to serve Chinese mapping when western models became the standard during the Republican period. Apart from this the maps enabled the Kangxi Emperor to negotiate boundaries with the Russian Empire who most likely accepted the Jesuit provenance of the maps as they would never have accepted traditional Chinese maps. In this document, a set of maps originally wood block printed in 1721 and later reprinted in facsimile by Walter Fuchs in 1941 [7] were used to study the map accuracy and develop a mosaic for a specific area that included parts of the five provinces of Shanxi, ShanGan (a composite province of what is today Shaanxi, Gansu and Ningxia), Sichuan, Henan and Huguang (another composite province of what is today Hubei and Hunan). The maps have been preserved, scanned at high resolution and made accessible to the public by the US Library of Congress [8]. The map projection parameters of the individual province maps, at near 1:1.94M scale, were estimated and used to re-project the maps and create presentations in Google Earth [12]. The mosaic components were cut out of the sheets and combined into the final mosaic product that is also available as a Google Earth presentation. The resulting presentations were not “warped” into modern geographic frameworks [9a] but maintain the original projection and geolocation.

The accuracy of the maps and the mosaic were investigated using the Gazetteer of mapping places provided by du Halde [3]. The places from the Gazetteer included in the Mosaic were then used to estimate the base accuracy of the Mosaic. In general, the error in the best mapped areas was about 6-8km in distance equivalents for Latitude and Longitude although in most places it was worse at about 8km for Latitude and 10km for Longitude in RMS error. The main errors, which tended to be extended into adjacent regions by the method used, were in Longitude with some large distortions of the order of 20km occurring in specific areas. Most of these were the result of inaccurate astronomical measurements and/or the sparse network of map control available to the Jesuit Brothers. But at the same time in the development of such surveys, the situation would have been little better for European maps. Longitude was to remain a problem for another 50 years [13,14]. As comparison, the Martini Maps [20] published in Europe from early Jesuit surveys in 1655 were investigated. The mapping was done in a similar way to the Kangxi maps except there was no ground survey and few astronomical estimates made for Longitude. Martini also provided a Gazetteer of coordinates for places and the accuracies can be compared with the Kangxi maps. The Kangxi maps represent an order of magnitude improvement with 100km total absolute errors and 50km standard deviations in the Martini maps changing to about 10km in each measure for the maps produced only 50 years later.

In the Mosaic, the RMS (total error) accuracy was about 8km in Latitude and 11km in Longitude. The Longitude errors occurred in specific areas which were analysed in more detail in this document. Within the mosaic there was significant variation in error. For

example along the Wei River the RMS errors (include bias error) were 4km for Latitude and 7km for Longitude and the Jialing 9km and 5km. But Longitude errors affected Han, Chang Jiang and Yellow River cases with the Han and Chang Jiang being 10km Latitude and 14km Longitude RMS in each case. But these are not really very bad errors in context. Because the (original) projection parameters for the Mosaic have been established it can easily be re-projected to (say) a Transverse Mercator with reference Longitude near its centre. In a future activity this projection can be treated as a square metric grid of the kind traditional Chinese maps attempt to realise. It is planned to use it as a collating base for a number of Chinese maps of various scale and extent that were produced between 1805 and 1825. Because the places on the Kangxi maps are annotated to garrison and fortress level it is hoped the map details will help significantly in this task. The accuracy of the Kangxi map Mosaic is sufficient for this task.

Similar exercises could be extended to other provinces or groups of provinces for more complete analysis of the maps. There seems to be a particularly significant bias that spreads through the south-east of the map. In this document it is conjectured the anchor point at Guangzhou may be the cause. However, to establish the truth of this, a study of the provinces surveyed together by the specific groups of Jesuit Brothers (Jiangxi, Guangdong and Guangxi in one set and Sichuan, Yunnan, Guizhou and Huguang in another) provide a sensible stratification. It is, however, simply a repeat of the methods used in this document. The maps of 1721 were apparently updated during the Qianlong period. Good copies of the later maps do not seem to be as easy to find but if some can be found a comparison between the maps would also be of value as would be the use of the same methods on good prints of the copper plate maps. Finally, the small impact made on traditional Chinese mapping has been discussed. Basically, the new maps seem to have provided little benefit for the needs of local Gazetteer maps or journey maps. However, it would have been quite possible for the two traditions to come together if relationships between projections and traditional coordinate systems had been better explained. Land maps can profitably use grid systems as presently represented by the Universal Transverse Mercator (UTM or Gauss-Kruger) system. Nested scales of map sheets combining the needs of mapping on a spheroidal Earth and those of the natural gridded framework preferred by Chinese for the local Gazetteers can be easily defined once free of the Sinusoidal format. However, as noted by Needham and Wang [17], such systems only developed as standard in the west in the 20th century and then (apparently) based on the needs of artillery and were realised as the Military Ordinance Survey maps that are used by hikers even now when GPS is everyone's survey tool.

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## 8. Appendix 1 Table of Proposed Anchor Point Locations

Province	du Halde Place	Chinese Province	Chinese Place (Maps)	Pinyin
Chan Tong	Laizhou Fu	山東	萊州府	Laizhou Fu
Chan Tong	Tengzhou fu	山東	登州府	Dengzhou Fu
Chen Si	Kia yu koan	陝西	嘉峪關	Jiayu Guan
Chen Si	Leang Zhou	陝西	涼州	Liang Zhou
Chen Si	Sou tcheou	陝西	肅州	Su Zhou
Chen Si	Si ngan fou	陝西	西安府	Xi'an Fu
Chen Si	Sining	陝西	西寧州	Xining Zhou
Ho Nan	Ho nan fou (Luoyang)	河南	河南府	Hehan Fu (Luoyang)
Ho Nan	Kai fong fou	河南	開封府	Kaifeng Fu
Kiang Nan	Nan King	江南	江寧府	Jiangning Fu (Nanjing)
Kiang Nan	Ngan king fou	江南	安慶府	Anqing Fu
Pe Tche Li	Tchang kia keou	北直隸	張家口堡	Zhangjia Kou Bao
Pe Tche Li	Peking	北直隸	京城	Jing Cheng (Beijing)
Pe Tche Li	Chan hai koan	北直隸	山海關	Shanghai Guan
Quang Tong	Quang tcheou fou	廣東	廣州府	Guangzhou Fu
Se Tchien	Ho tcheou	四川	合州	He Zhou
Se Tchien	Ta Tsien lou	四川	打箭爐	Dajian Lu
Tche Kiang	Ning po fou	浙江	寧波府	Ningbo Fu
Tche Kiang	Hang Tcheou Fou	浙江	杭州府	Hangzhou Fu
Yun Nan	Li kiang fou	雲南	麗江土府	Lijiang Tu Fu

## 9. Appendix 2 Table of Errors in Anchor Points relative to ChinaW Points

Anchor Points								
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	Du Halde	Du Halde	ChinaW	ChinaW		1 deg Y (km)=		
Name	Lat	Lon	Lat	Lon	Lat_Err	Lon_Err	y Err	x Err
Laizhou Fu	37.160	120.145	37.175	119.938	0.015	-0.207	1.716	-18.380
Dengzhou Fu	37.807	120.993	37.805	120.741	-0.003	-0.252	-0.301	-22.155
Jiayu Guan	39.806	98.764	39.796	98.283	-0.009	-0.481	-1.022	-41.080
Liang Zhou	37.983	102.718	37.927	102.635	-0.056	-0.083	-6.273	-7.278
Su Zhou	39.761	99.034	39.745	98.511	-0.016	-0.523	-1.798	-44.723
Xi'an Fu	34.260	108.818	34.267	108.944	0.007	0.127	0.734	11.627
Xining Zhou	36.656	101.718	36.609	101.784	-0.047	0.066	-5.193	5.914
Henan Fu	34.721	112.379	34.665	112.383	-0.056	0.004	-6.178	0.351
Kaifeng Fu	34.868	114.468	34.785	114.343	-0.083	-0.124	-9.182	-11.356
Nanjing	32.075	118.702	32.053	118.769	-0.022	0.067	-2.495	6.302
Anqing Fu	30.619	116.988	30.512	117.035	-0.107	0.047	11.914	4.476
Zhangjia Kou	40.860	114.846	40.791	114.886	-0.068	0.040	-7.613	3.362
Beijing	39.917	116.393	39.900	116.392	-0.017	-0.001	-1.853	-0.096
Shanghai Guan	40.042	119.761	40.010	119.770	-0.031	0.009	-3.500	0.786
Guangzhou	23.183	112.868	23.135	113.256	-0.048	0.388	-5.354	39.686
Hezhou	30.140	106.318	30.004	106.260	-0.136	-0.058	15.129	-5.596
Dajian Lu	30.140	101.765	30.053	101.958	-0.087	0.194	-9.724	18.632
Ningbo Fu	29.920	121.348	29.866	121.543	-0.054	0.195	-5.969	18.774
Hangzhou Fu	30.339	120.044	30.294	120.169	-0.045	0.125	-4.978	11.986
Lijiang Tu Fu	26.860	100.373	26.875	100.236	0.015	-0.137	1.627	-13.613

## 10. Appendix 3 Table of Martini, du Halde and ChinaW Places for error comparison

Map	Martini			du Halde			ChinaW		
CH Name	"Name"	"Lat"	"Lon"	"Name"	"Lat"	"Lon"	"Name"	"Lat"	"Lon"
廣州府	Quangcheu	23.2500	112.3577	Quang tcheou fou	23.1828	112.8680	Guangzhou Fu	23.1346	113.2561
永州府	Yungcheu	26.7000	110.3910	Yong tcheou fou	26.1400	111.4966	Yongzhou Fu	26.2103	111.6126
衡州府	Hengcheu	27.8000	111.1743	Heng tcheou fou	26.9200	112.2993	Hengzhou Fu	26.9016	112.5970
宝慶府	Paoking	27.7167	110.3077	Pao king fou	27.0600	111.2716	Baoqing Fu	27.2493	111.4749
長沙府	Changxa	28.8333	111.2910	Tchang cha fou	28.2000	112.6957	Changsha Fu	28.1982	112.9781
辰州府	Xincheu	29.1000	109.8077	Tching tcheou fou	28.3736	110.0577	Chenzhou Fu	28.4582	110.3970
馬湖府	Mahu	29.0833	103.0743	Ma hou fou	28.5167	104.2243	Muhu Fu	28.6571	104.1610
敘州府	Siucheu	29.2167	103.9577	Soui tcheou fou	28.6400	104.6766	Xuzhou Fu	28.7748	104.6164
常德府	Changte	29.6333	110.2577	Tchang te fou	29.0167	111.3624	Changde Fu	29.0348	111.6913
岳州府	Yocheu	30.0833	111.7243	Yo tcheou fou	29.4000	112.8229	Yuezhou Fu	29.3713	113.0977
重慶府	Chungking	30.3833	106.0077	Tchong king fou	29.7000	106.6160	Chongqing Fu	29.5600	106.5527
寧波府	Ning'po	29.6667	121.1577	Ning po fou	29.9200	121.3480	Ningbo Fu	29.8663	121.5427
杭州府	Hangcheu	30.4500	119.5577	Hang Tcheou Fou	30.3389	120.0438	Hangzhou Fu	30.2941	120.1686
荊州府	Kingcheu	30.8333	110.5910	King tcheou fou	30.4444	111.9966	Jingzhou Fu	30.3504	112.1908
黃州府	Hoangcheu	31.3833	113.5577	Hoang tcheou fou	30.4400	114.7313	Huangzhou Fu	30.4470	114.8655
漢陽府	Hanyang	30.8333	112.6743	Han yang fou	30.5772	114.0846	Hanyang Fu	30.5536	114.2634

武昌府	Vuch'ang	31.0000	113.1243	Ou tchang fou	30.5806	114.1410	Wuchang Fu	30.5607	114.3057
城都府	Chingtu	30.7833	103.7577	Tching tou fou	30.6781	104.0910	Chengdu Fu	30.6504	104.0780
順慶府	Xunking	31.2833	105.7243	Chun king fou	30.8200	106.0410	Shunqing Fu	30.7990	106.0807
夔州府	Queicheu	31.0500	108.3410	Koei tcheou fou	31.1600	109.4993	Kuizhou Fu	31.0547	109.5248
安陸府	Chingtien	31.5833	111.0577	Ngan lo fou	31.2000	112.3910	An'lu Fu	31.1698	112.5893
德安府	Tegan	31.8500	112.2243	Te ngan fou	31.3000	113.5438	De'an Fu	31.2637	113.6889
保寧府	Paoning	31.8833	105.3910	Pao king fou	31.5400	105.8910	Baoning Fu	31.5831	105.9695
襄陽府	Siangyang	32.4667	110.8410	Siang yang fou	32.1000	112.0121	Xiangyang Fu	32.0236	112.1596
江寧府	Kiangning	32.6667	117.8243	Nan King	32.0750	118.7021	Jiangning Fu	32.0526	118.7690
龍安府	Lunggan	32.7500	104.2243	Long ngan fou	32.3667	104.5632	Long'an Fu	32.4132	104.5297
鄖陽府	Chingyang	33.0000	109.5243	Yuen yang fou	32.8222	110.7774	Yunyang Fu	32.8340	110.8140
南陽府	Nanyang	33.8833	111.1410	Nan yang fou	33.1042	112.4924	Nanyang Fu	33.0017	112.5355
汝寧府	Iuning	33.8833	113.4577	Yu nhing fou	33.0167	114.2660	Runing Fu	33.0075	114.3460
漢中府	Hanchung	34.3333	106.5243	Han tchong fou	32.9861	107.1229	Hanzhong Fu	33.0765	107.0352
西安府	Sigan	35.8333	108.0910	Si ngan fou	34.2600	108.8160	Xi'an Fu	34.2666	108.9442
歸德府	Queite	35.1667	114.8577	Koue te fou	34.4778	115.7660	Guide Fu	34.3851	115.6075
鳳翔府	Fungciang	36.8333	107.1410	Fong tsiang fou	34.4200	107.4091	Fengxiang Fu	34.5225	107.3859
河南府	Honan	35.6333	111.3077	Ho nan fou	34.7208	112.3771	Hehan Fu	34.6653	112.3826
開封府	Caifung	35.8333	113.4910	Kaï fong fou	34.8681	114.4660	Kaifeng Fu	34.7855	114.3433
玕昌府	Cungch'ang	36.8500	104.8243	Kong tchang fou	34.9400	104.6410	Gongchang Fu	35.0071	104.6356
懷慶府	Hoaking	36.1667	111.8077	Hoai king fou	35.1094	112.9160	Huaiqing Fu	35.0893	112.9370
臨洮府	Linyao	36.7833	104.2910	Ling tao fou	35.3600	103.8910	Lintao Fu	35.3791	103.8579



衛輝府	Gueihoei	36.5000	113.0577	Oue kiun fou	35.4611	114.1827	Weihui Fu	35.4104	114.0703
平涼府	Pingleang	37.2000	106.7077	Ping leang fou	35.5800	106.5910	Pingliang Fu	35.5359	106.6857
慶陽府	Kingyang	37.4500	107.2910	King yang fou	36.0500	107.6243	Qingyang Fu	36.0052	107.8760
平陽府	Pingyang	37.3167	110.4243	Ping yang fou	36.1000	111.4660	Pingyang Fu	36.0780	111.5154
彰德府	Changte	37.0000	112.9577	Tchang te fou	36.1222	114.4160	Zhangde Fu	36.0983	114.3455
潞安府	Lugan	37.2167	112.0577	Lou ngan fou	36.1200	112.9160	Lu'an Fu	36.1835	113.1035
延安府	Lengan	37.6167	108.0577	Yen ngan fou	36.7056	109.3160	Yan'an Fu	36.5918	109.4699
汾州府	Fuencheu	38.1667	110.3910	Fuen tcheou fou	37.3200	111.6160	Fenzhou Fu	37.2634	111.7790
太原府	Taiyuen	38.5500	111.8077	Tai yuen fou	37.8917	112.4660	Taiyuan Fu	37.7276	112.4789
大同府	Taitung	40.3333	112.2243	Tai tong fou	40.0950	113.1910	Datong Fu	40.0928	113.2964

## 11. Appendix 4 Table of Errors between the Martini and du Halde maps and ChinaW places

Map	Martini	du Halde	ChinaW	Deg	Deg	km	km
CH Name	"Name"	"Name"	"Name"	Lat_Err	Lon_Err	y Err	x Err
廣州府	Quangcheu	Quang tcheou fou	Guangzhou Fu	0.1154	-0.8984	12.829	-91.864
永州府	Yungcheu	Yong tcheou fou	Yongzhou Fu	0.4897	-1.2216	54.448	121.865
衡州府	Hengcheu	Heng tcheou fou	Hengzhou Fu	0.8984	-1.4227	99.896	141.078
宝慶府	Paoking	Pao king fou	Baoqing Fu	0.4674	-1.1673	51.967	115.390
長沙府	Changxa	Tchang cha fou	Changsha Fu	0.6351	-1.6871	70.624	165.336
辰州府	Xincheu	Tching tcheou fou	Chenzhou Fu	0.6418	-0.5893	71.366	-57.609
馬湖府	Mahu	Ma hou fou	Muhu Fu	0.4262	-1.0866	47.390	106.028
敘州府	Siucheu	Soui tcheou fou	Xuzhou Fu	0.4418	-0.6587	49.130	-64.199
常德府	Changte	Tchang te fou	Changde Fu	0.5985	-1.4336	66.553	139.379
岳州府	Yocheu	Yo tcheou fou	Yuezhou Fu	0.7121	-1.3733	79.179	133.078
重慶府	Chungking	Tchong king fou	Chongqing Fu	0.8234	-0.5450	91.554	-52.713
寧波府	Ning'po	Ning po fou	Ningbo Fu	-0.1996	-0.3850	-22.200	-37.123
杭州府	Hangcheu	Hang Tcheou Fou	Hangzhou Fu	0.1559	-0.6110	17.333	-58.659
荊州府	Kingcheu	King tcheou fou	Jingzhou Fu	0.4829	-1.5998	53.695	153.508
黃州府	Hoangcheu	Hoang tcheou fou	Huangzhou Fu	0.9363	-1.3078	104.117	125.370
漢陽府	Hanyang	Han yang fou	Hanyang Fu	0.2797	-1.5891	31.104	152.164
武昌府	Vuch'ang	Ou tchang fou	Wuchang Fu	0.4393	-1.1814	48.849	113.117
城都府	Chingtu	Tching tou fou	Chengdu Fu	0.1329	-0.3203	14.783	-30.643
順慶府	Xunking	Chun king fou	Shunqing Fu	0.4843	-0.3564	53.857	-34.040
夔州府	Queicheu	Koei tcheou fou	Kuizhou Fu	-0.0047	-1.1837	-0.523	112.762
安陸府	Chingtien	Ngan lo fou	An'lu Fu	0.4135	-1.5316	45.980	145.719

德安府	Tegan	Te ngan fou	De'an Fu	0.5863	-1.4646	65.192	139.206
保寧府	Paoning	Pao king fou	Baoning Fu	0.3002	-0.5785	33.380	-54.795
襄陽府	Siangyang	Siang yang fou	Xiangyang Fu	0.4430	-1.3186	49.265	124.313
江寧府	Kiangning	Nan King	Jiangning Fu	0.6141	-0.9447	68.285	-89.029
龍安府	Lunggan	Long ngan fou	Long'an Fu	0.3368	-0.3054	37.455	-28.667
鄖陽府	Chingyang	Yuen yang fou	Yunyang Fu	0.1660	-1.2897	18.458	120.495
南陽府	Nanyang	Nan yang fou	Nanyang Fu	0.8817	-1.3945	98.036	130.040
汝寧府	luning	Yu nhing fou	Runing Fu	0.8758	-0.8883	97.384	-82.835
漢中府	Hanchung	Han tchong fou	Hanzhong Fu	1.2569	-0.5109	139.756	-47.599
西安府	Sigan	Si ngan fou	Xi'an Fu	1.5667	-0.8532	174.212	-78.405
歸德府	Queite	Koue te fou	Guide Fu	0.7816	-0.7498	86.912	-68.804
鳳翔府	Fungciang	Fong tsiang fou	Fengxiang Fu	2.3108	-0.2449	256.951	-22.438
河南府	Honan	Ho nan fou	Hehan Fu	0.9681	-1.0750	107.643	-98.313
開封府	Caifung	Kaī fong fou	Kaifeng Fu	1.0479	-0.8523	116.516	-77.838
玁昌府	Cungch'ang	Kong tchang fou	Gongchang Fu	1.8429	0.1887	204.917	17.189
懷慶府	Hoaking	Hoai king fou	Huaiqing Fu	1.0774	-1.1293	119.798	102.753
臨洮府	Linyao	Ling tao fou	Lintao Fu	1.4042	0.4331	156.139	39.262
衛輝府	Gueihoei	Oue kiun fou	Weihui Fu	1.0896	-1.0126	121.160	-91.768
平涼府	Pingleang	Ping leang fou	Pingliang Fu	1.6641	0.0220	185.045	1.988
慶陽府	Kingyang	King yang fou	Qingyang Fu	1.4448	-0.5850	160.659	-52.624
平陽府	Pingyang	Ping yang fou	Pingyang Fu	1.2386	-1.0911	137.729	-98.057
彰德府	Changte	Tchang te fou	Zhangde Fu	0.9017	-1.3878	100.260	124.692
潞安府	Lugan	Lou ngan fou	Lu'an Fu	1.0331	-1.0458	114.879	-93.861
延安府	Lengan	Yen ngan fou	Yan'an Fu	1.0249	-1.4122	113.959	126.083
汾州府	Fuencheu	Fuen tcheou fou	Fenzhou Fu	0.9033	-1.3880	100.438	122.835
太原府	Taiyuen	Tai yuen fou	Taiyuan Fu	0.8224	-0.6713	91.448	-59.036
大同府	Taitung	Tai tong fou	Datong Fu	0.2405	-1.0721	26.742	-91.194