ARTICLE

Projection, Scale, and Accuracy in the 1721 Kangxi Maps

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ABSTRACT

The Kangxi Emperor employed Jesuit brothers (1708–18) to produce maps of the provinces of China using a combination of Western and Chinese survey methods. The maps were completed by 1721. They were sent back to Europe and became the basis for maps of China produced by Jean Baptiste Bourguignon d'Anville in 1735. The main changes from traditional Chinese mapping were to use latitude and longitude as primary coordinates, map them using a spherical projection, and use astronomical measurements of latitude and longitude to establish baselines. Changes in latitude and longitude were found using traditional metric survey and relationships between distance north–south and latitude and distance east–west and longitude to convert to degrees. A digitized facsimile of the 1721 map series is available from the US Library of Congress. In this article, the digitized images were used to reconstruct the parameters of the sinusoidal projection, establish scale, and re-project and mosaic the maps into various forms for presentation. The accuracy of five of the province maps is discussed in detail. It is found that poor astronomical measurements of longitude are the most serious issue for the maps, but that apart from areas distorted by the poor longitude estimates, the accuracy was commensurate with that of European land maps of the time.

Keywords: Kangxi maps, map projections, Jesuit surveys, Chinese maps, map accuracy

RÉSUMÉ

L'empereur Kangxi a fait appel aux frères jésuites (1708–1718) pour tracer les cartes des provinces de Chine à l'aide d'une combinaison de méthodes d'étude occidentales et chinoises. Ces cartes ont été achevées en 1721. Elles ont été réacheminées en Europe où elles ont, par la suite, servi de base à la production de cartes de la Chine par Jean Baptiste Bourguignon d'Anville, en 1735. Les principales innovations de ces cartes par rapport aux cartes traditionnelles chinoises étaient l'utilisation de la latitude et de la longitude comme coordonnées principales, la représentation par projection sphérique et l'utilisation de mesures astronomiques de latitude et de longitude pour établir des bases de référence. Les variations de latitude et de longitude étaient déterminées à l'aide d'une étude métrique classique et des relations entre la distance nord-sud et la latitude et entre la distance est-ouest et la longitude, aux fins de conversion en degrés. Un fac-similé numérisé de la série de cartes de 1721 peut être consulté à la Bibliothèque du Congrès des États-Unis. L'auteur utilise ces images numérisées pour reconstruire les paramètres de la projection sinusoïdale, établir l'échelle et reprojeter et assembler en mosaïque ces cartes en les présentant sous diverses formes. La précision de cinq des cartes des provinces fait l'objet d'une analyse détaillée. L'exercice révèle que le problème le plus sérieux que présentent ces cartes tient aux déficiences des mesures astronomiques de longitude mais que, outre les distorsions régionales attribuables aux piètres estimations de longitude, la précision de ces cartes se compare à celle des cartes terrestres européennes de l'époque.

Mots clés : cartes chinoises, cartes Kangxi, études des jésuites, précision cartographique, projections cartographiques

Introduction

In 1718, on the basis of 10 years of field surveys covering the Qing Empire, a group of French Jesuit Brothers and Chinese surveyors produced a set of province and region maps for the Kangxi Emperor. The maps were the most accurate cartographic mapping of China ever carried out at that time and were probably at least similar in accuracy to European maps of the time. Chinese in general knew little about these maps until the twentieth century. However, the information became available in Europe quite soon after 1721 and formed the basis for Jean Baptiste Bourguignon d'Anville's *Nouvel atlas de la Chine, de la Tartarie Chinoise et du Thibet* (or *New atlas of China, Chinese Tartary and Tibet*; d'Anville 1737). Most of the maps of inner China at this time were reproduced by d'Anville without change, as described in detail by Mario Cams (2014). A mosaic of all the maps was assembled by d'Anville and published by Jean Baptiste du Halde (1735) as part of a monumental "encyclopaedia" of China as it was known from the missionary work of the Society of Jesus. A comprehensive description of the Jesuit mapping activity can be found in du Halde's book, for which French (du Halde 1735) and English (Cave 1741) versions are available.

The maps of individual provinces and regions were on the sinusoidal projection, which is sometimes called the Sanson or Mercator–Sanson map projection. Nicholas Sanson (1600–67) was at one time Royal Cartographer of France and used the projection ca. 1650. The maps have Beijing as standard meridian. The Kangxi province maps drafted in China all used this simple and mathematically well-founded projection (Pearson 1990) and the Beijing standard meridian, as did the Martini maps of China approximately 60 years earlier (Martini 1655). However, it is only recently that the specific form has become established in the literature of these maps. Cordel Yee (1994), in his comprehensive discussion of the Kangxi maps, refers to the projection as "trapezoidal", and even at the time when du Halde wrote his work, the specific projection was not clear. For example, du Halde (1735) and d'Anville (1737) referred to the projection simply as a "general" map projection ("*une projection Générale*"; du Halde 1735, Preface, p. lvj) which was translated in Cave (1741) as a "plain" projection. Edward Cave, in his Preface to the English translation of du Halde (Cave 1741), went further to complain that

> Being exhibited on the plain Projection with inclining Meridians, Countries are thrown out of their natural Figure and Proportion: Whence this Deformity, tho' scarce discernible in the Maps of Pe che li, Shan tong, Kyang nan and Kyang si, thro' which the Meridian of Peking passes, is yet very perceptible in those of Shensi, Se chwen and Yun nan, which lye farthest from it.

However, Wang Qianjin (Wang 1991) has shown that the projection is unequivocally sinusoidal. The maps he used in his investigation were the copperplate edition, engraved by Jesuit Brother Matteo Ripa and usually referred to (in English) as "Complete map of the imperial domain." A set of the plates was found in the Imperial Palace at Shenyang in 1921. Because it was until that time unknown, it has since also been known as the "Kangxi Secret Map." Using the copperplates, because there was no paper stretch or fold distortions, the geometry was true to the original, allowing Wang Qianjin to reach his conclusions on the basis of precise measurements. Wang Qianjin also concluded that the scale of the copperplate map was 1:1.4 million.

In 1941, a German missionary, Walter Fuchs (Regis and others 1941), published facsimile prints of an original 1721 wood block printed set of 35 Province and region maps essentially identical to the maps sent back and published in Europe by d'Anville, as described by Cams (2014). A copy of these maps is held by the US Library of Congress, where they have recently been scanned at high resolution into accessible digital images (Geography and Map Division of the Library of Congress). The grid has 0.5° spacing and the maps include places down to township and garrison level. The present study has concluded that the maps are at 1:1.94 million scale, which makes them a little coarser than the map scale quoted for the copperplate maps. Yee (1994) tackled the fairly complex story of the various early maps. He concluded that an initial set of 28 woodblock printed maps was apparently delivered in 1719, the copperplate version described above with 41 sheets was next, and a revised version of 35 sheets appeared in 1721 and was sent to Europe. The latter was almost certainly the version reprinted by Fuchs, scanned by the US Library of Congress, and used here.

The maps of the 35 sheets provide an important historical snapshot of the borders, areas of expansion, province amalgamations, names of places, and other map information as it existed near the end of the Kangxi Emperor's reign. The 15 "inner" provinces were similar in extent to how they were at the end of the Ming. Their western and northern borders differ significantly from those of today, and there are also three composites of today's provinces. These were a combination of present-day Shaanxi, Gansu, and Ningxia called here ShanGan, another of today's Hubei and Hunan called Huguang, and a province called Jiangnan which is roughly present-day Jiangsu and Anhui. In this document, the 5 individual province maps for ShanGan, Shanxi, Henan, Sichuan, and Huguang from among the 15 are analyzed in detail for scale and accuracy. These five maps comprise eight modern central and western provinces. The specific projection parameters for the five provincial maps were established based on grid crossings and equality of borders. They were then used to reproject the Jesuit maps to a suitable form for import to software such as Google Earth, as well as to form a mosaic of the five province maps. The maps were not warped using information from modern maps, as was done by Alexander Akin and David Mumford (Akin and Mumford 2012), but rather reconstructed in their original form. This document then evaluates the scale and accuracy of this group of five of the original maps in terms of modern map information and measures.

There are occasions in the article when Chinese names and phrases need to be used. This article uses pinyin romanization and simplified form characters in most cases. However, where names of provinces or places refer directly to those on the 1721 maps, the full form characters are selected to be as close as possible to those drawn on the maps – which are sometimes archaic characters. The pinyin names of places from the map also separate the names and the designators, as at the time the designations (mostly administrative) were numerous and important. For example, 榆林衛 will be written here in pinyin as Yulin Wei, since the third character represents a type and level of military post. Sometimes today the old designators have become a part of the modern name and so are not separated in pinyin.

The Background and Technology of the Jesuit Survey of China

SUMMARY OF THE ACTIVITIES OF THE JESUIT BROTHERS

Most of the information summarized here has been gleaned from du Halde (du Halde 1735; Cave 1741), Yee (1994), Cams (2014), Han Qi (Han 2006) and Joseph Needham and Wang Ling (Needham and Wang 1971). The proposal to map the complete extent of the Empire arose after successful field trials of the survey methods had been carried out and survey measurements had been standardized. The work plan was based on teams consisting of relatively few Jesuit Brothers working with Chinese surveyors to survey transects throughout the country. The complete project occurred in roughly three stages: (i) develop the Great Wall as a baseline transect; (ii) map throughout the far north (Tartary) to help establish boundaries with Russia; and (iii) map the 15 main "inner" provinces of China. Maps of Korea and Tibet were included in the output map set but were not surveyed under direction of the Jesuit Brothers. Re-projected images of the five provinces used here have been converted to KML super-overlays for input to Google Earth or similar software as illustrated under *Forming Mosaic Images*. A super-overlay structure allows an image to be displayed at different levels of detail with zoom from general to particular scales so that only small amounts of data are needed at each stage. Super-overlays of all of the images discussed here have been made available at the author's Web site (Jupp n.d.).

In the following paragraphs, **bolded** text draws attention to places and areas relevant to the five provincial map sheets. In these maps (Geography and Map Division of the Library of Congress), the **Great Wall** is mapped in considerable detail from Jiayu Guan across **ShanGan**, **Shanxi**, and the Bei Zhili to the sea at Shanhai Guan. The detail includes gates, forts, and associated administrations along its full extent. In later surveys in northern China, including **ShanGan** and **Shanxi**, the Great Wall acted as a horizontal baseline. In du Halde (du Halde 1735; Cave 1741), it is recorded that the Great Wall was mapped between July 1708 and January 1709, and in May 1709 a survey team went north into the country of the Manchu (Manchuria). This area north extended from the 40th to the 45th parallel (with measure places extending up to 51° north). As described by Elman (2005), the Russians had for some time been active on China's northern boundaries, and accurate surveys (represented as maps Russians would accept) were needed to negotiate borders. After the Brothers arrived back in Beijing, they were tasked to complete a map of the direct rule area around Beijing, or the Bei Zhili (北直隸). They were then sent back north and northwest to the western borders threatened by Russian advances at a latitude above 47°. This stage was completed by the end of 1710, when attention shifted to what is sometimes called "inner" China.

From 1711, the surveys were divided between different groups as the work spread south and west throughout China from the base in Beijing. In the first split, one team (Brothers Regis and Cardoso) was sent to map Shandong. The other team (Brothers Jartoux, Fridelli, and Bonjour) travelled beyond the Great Wall into Xinjiang as far as Hami and then mapped back, re-entering China at the Northwestern Jiayu Guan entrance of the Great Wall.

In 1712, the Emperor asked (du Halde 1735) if some others who were skilled in the survey methods could be found to join the effort, and an additional four Brothers were enlisted. The major actions were as follows:

(1) Fr. Cardoso went to Shanxi, where a Fr. de Tartre was stationed, and with local surveyors mapped the provinces of **ShanGan** and **Shanxi**. The two brothers then went south to map the provinces of Jiangxi, Guangdong, and Guangxi.

(2) Brothers Regis, de Mailla, and Henderer mapped through **Henan**, Jiangnan (present-day Anhui and Jiangsu), Zhejiang, and Fujian.

(3) Brothers Fridelli and Bonjour were sent to map the provinces of **Sichuan** and Yunan. However, Fr. Bonjour died near the border with Burma and Fr. Fridelli also fell ill. Fr. Regis was later sent to complete the map of Yunnan. Fr. Fridelli had recovered and together they went on to map Guizhou and **Huguang**.

When the accuracy of the maps is considered later, it is necessary to take account of the conditions that prevailed, the teams involved, and the routes followed in the various mapping missions as described above. The complete set of surveys was finished by January 1717. The next year was apparently spent collating the data and developing the final products to present to the Emperor (du Halde 1735; Cave 1741).

ASTRONOMICAL MEASUREMENTS AND THE METHOD OF TRIANGLES

From du Halde's descriptions (du Halde 1735; Cave 1741), it appears that soon after the French Jesuit Brothers arrived, a large number of places in China were measured using a range of recently developed astronomical methods (Konvitz 1987). However, the astronomical measurements were not always used during the intense surveys in "inner"

China after 1710. The problem was that under the difficult conditions of the field surveys, they could be subject to large errors. Rather, some of the existing places where Jesuit astronomers had spent more time in the past to obtain accurate longitude data were used for baseline control. During the surveys, astronomical measurements of latitude were more often made. These were most likely based on the declination-corrected elevation of the Pole Star with cross checks using the position of the sun at noon. As time was short and weather not always co-operative, these measurements were not always accurate. However, combining survey methods and astronomical measurements likely improved the latitude estimates significantly.

The primary survey technique was traditional surveying with chain (distance), compass (direction), and staff (height). If the route from one place to another was carefully mapped in short stages, estimating distances, directions, and changes in altitude, then, after adjustments to the data, the Theorem of Pythagoras could be used to resolve the length of the route "as the bird flies" (*feiniao*, 飞鸟) into an incremental distance north or south and an incremental distance east or west between the places. If the distances involved are less than (maybe) 200 km as the bird flies, the earth can be assumed to be locally flat and Euclidean geometry to prevail. The change in latitude and the change in longitude can then be estimated if the relationships between distance over the earth and change in angles in the north–south direction (change in latitude) and in the east–west direction (change in longitude) are known. Starting from a place where the latitude and longitude are known, the corresponding latitudes and longitudes of other places along the survey route can then (in principle) be calculated and adjusted when accurate astronomical measurements are made. The survey method itself does not result in absolute latitude and longitude, but rather in increments of latitude and longitude. So, apart from places on a direct survey line from Beijing, some independent astronomical measurements were essential as "anchor points." Since latitudes were measured regularly, it follows that if the surveys also included enough places where accurate longitudes were measured or known, then corrections could be made based on these and overall accuracy could be high.

Chinese surveyors already knew effective methods for plane survey to measure distance "as the bird flies," and the principles were outlined in ancient times by Pei Xiu (裴秀) during the Jin Dynasty (265–420 CE). A part of Pei Xiu's work has been translated from a modern biography in the Discussion section. Traditional Chinese mapping took no account of the curvature of the Earth, but if the areas mapped were smaller than (say) 400 by 400 km, it was not a problem unless the maps needed to be joined in a larger mosaic. It is therefore likely that forming competent Chinese teams to undertake the surveying was not a problem. As outlined by Han Qi (Han 2006), before the work started, the Kangxi Emperor insisted that princes of the realm learn the methods used by the Jesuit Brothers. Han Qi (Han 2006) describes the trials made by the court before the main mapping survey was commissioned. He quotes from a record of the instructions given to the Princes by the Kangxi Emperor:

> On the whole, the method used is mostly geometrical triangulation. Although the name *sanjiaoxing* [三角形, triangle method] did not exist before, the mathematical method must always have it as its basis. For instance, the method of Gou-gu [Pythagorean theorem] is derived from triangle, and this method was passed down from ancient times. However, it was not recorded in books. Therefore people do not know its origin.

The Emperor equates the triangle method with the principles of Pei Xiu rather than present-day survey triangulation. In doing this, he spoke from personal interest, as he had published a treatise on the "Derivation of Triangles" in 1703 (Han 2016). Therefore, because resolving distance as the bird flies into increments north–south and east–west was well known, it was only in the step from traditional surveying to the astronomical system of latitude and longitude and the spherical earth that the activity moved out of areas already familiar to Chinese. Sightings to towers or other landmarks in nearby towns which had previously been measured were also used for "true" triangulation to close some of the triangles. In addition, occasionally the surveys would pass the same place again, allowing survey cross checking. But ultimately, it was the resolved incremental distances as the bird flies that formed the base data. This simplification was almost certainly the reason that the Jesuit Brothers were able to map the whole of China in 10 years, whereas it took about 70 years (1668–1744; Konvitz 1987) using survey by baseline and dense triangulation for France to be mapped to a similar level of detail.

STANDARDIZING THE LI

One of the most critical needs for the survey was to establish the relationships between degrees of latitude and longitude and distance on the ground. To be sure of the measured distances on the ground, it was also critical to standardize the measurements in Chinese units of distance and in terms of the measuring devices used in the field. All of these were addressed before the surveys began, in experiments carried out in company with the Kangxi Emperor or Qing princes. In Europe at the time, the figure of the Earth was still assumed to be a sphere, and the Jesuit Brothers would have known that on a sphere, the distance across the surface (of the sphere) on a meridian due north (or south) between parallels for a 1° change of latitude is everywhere the same and can be written as

$$h_y = R \times \frac{\pi}{180}.$$

In this equation, R is the radius of the spherical earth and $\pi/180$ is an angle of 1° in radians. The spherical radius used here for the sinusoidal projection was 6,371,007 m. This is the radius of the sphere with the same centre and area as the WGS84 spheroid (the "authalic" sphere for the spheroid). It follows that the distance on this sphere is 111.1947 km. The Jesuit Brothers would also have known that for the distance on a parallel corresponding to a 1° change in longitude, the formula changes with latitude and is

$$h_x = \cos\phi \times h_y.$$

In this equation, ϕ is latitude, so at the parallel of Beijing (taken here as 39.16667° North, which at the centre of the Forbidden City), the distance is $h_x = 85.2842$ km. As neither the Chinese surveyors nor the Jesuit Brothers used kilometres, it was necessary that the measure of these distances be Chinese li. But the li has never been well standardized and has varied in length equivalent to modern western standards quite considerably, both in time and place in China, over thousands of years (Qiu 2002), with there often being both a "short" and a "long" li in use at any one time and any one place.

In field experiments carried out by the court in the company of the Jesuit Brothers, the distance north corresponding to a 1° change in latitude (which can be determined by observing the sun at noon and/or the Pole Star altitude at night) was established to be 200 li exactly. How this came about is described a translated manuscript by Han Qi (Han 2006) quoting a court official, Li Guangdi (Li 1995), as follows:

> His Majesty arrived at Dezhou during his Southern inspection. [...] As calendar experts described, one degree in the sky, [is] equivalent to 250 li on ground level. Although I have not surveyed precisely, I feel that the distance should be 250 li. At present I have asked San-a-ge [三阿哥, third child] to carefully measure the distance from Beijing. San-a-ge's mathematical skills are extremely refined. Now at Dezhou, albeit a little inclined to the East, Gou-gu method [勾股, i.e., the Pythagorean theorem] is used to measure, making use of pegs and chunks to note the distance. Imprecise measurements will not happen any longer. Upon return to Beijing on the 21st, the Emperor said to [my] master: "San-a-ge has made the measurement, which means: one degree in the sky is exactly 200 li on ground level." My master said: "This is so because the system used was of eight Cun of Zhou dynasty's Chi, resulting that 250 li equals one degree.

By applying this to the distance above in kilometers, it is found that the li values corresponding to 200 and 250 li to 1° on the sphere are 555.98 and 444.78 m, respectively. The Emperor chose a "long" li, which was then a new standard for the li. The arguments based on historical records were most likely necessary for it to be accepted by the existing authorities. In the Qing period generally, 1 chi (尺) was 10 cun (寸), 1 bu (步) was 5 chi, and 1 li (里) was 300 bu or 360 bu, giving a short and long li (Qiu 2002). Despite this, a chi varied from place to place and application to application. It seems that, despite the explanations in terms of variations in traditional definitions of the relation between the cun and chi, what had happened was that a new standard had been created for the chi, bu and li and used for the survey. Provided the Jesuit Brothers' surveys used chains in the field to measure distances in terms of this (new standard) long li and any chains obtained locally were also calibrated to the standards, the survey had all that it needed.

Definitions of short and long li certainly had a close correspondence to short and long definitions of the French league at the same time. Corresponding to the chi was the French foot, or pied, which was 0.3248 m or 1.066 Imperial feet. Before the Revolution, the league had several definitions. Two were related to the length of a great circle arc of 1°. There was the "League of 25 to a degree" at 2282 toise (1 toise being 6 pied), and another established by the Paris Academy, which Fr. Regis reported as being 2853 toise, for which there were 20 to a degree of a great circle arc. So the short league is 10 short li and the Long League is 10 long li. This was no coincidence. The Brothers could then also happily convert new standard li into French leagues. Fr. Regis called the French long league the "marine league."

Calibrating the Map Projections

PROVINCE MAP SHEETS

The base materials are the set of maps originally printed by Walter Fuchs (Regis and others 1941) and now held by the US Library of Congress (Geography and Map Division of the Library of Congress). As noted above, 5 of the 35 maps have been specifically selected and studied in this article.

The five maps were scanned consistently by the US Library of Congress at 400 dpi (dots per inch) and square with north at top. There were some local distortions due to paper stretch and folds being too strong for the sheets to lie completely flat. Small remaining rotations from north were removed in Photoshop and the frame was trimmed to the neat line plus a margin to retain the latitude and longitude annotations. The image data (as TIFF files) were saved as grey scale and changed so that zero did not occur. Later, "zero" is used to represent null data and interpreted as "invisible" for mosaicking and display. The scan of the ShanGan province showing the Great Wall on its northern boundary is included here in its native projection as Figure 1 to indicate the typical base form.

The effects of the map folds are obvious in this image. The grid lines are latitude and longitude at 0.5°, spacing with longitude as difference from the Beijing meridian.

Table 1. Five provinces map sheets with Fuchs names, Chinese map names, Du Halde names, and geographic bounds

Map #	Fuchs Name	Chinese Map Name	Du Halde Name	Min_Lat	Max_Lat	Min_Lon	Max_Lon
24	Shensi (–Kansu)	陝西全圖	Chen Si	31.5000	40.0000	98.3910	111.8910
23	Shansi	山西全圖	Chan Si	34.5000	41.0000	109.8910	115.3910
25	Honan	河南全圖	Ho Nan	31.0000	37.5000	109.8910	116.8910
34	Szechuan	四川全圖	Se Tchuen	25.5000	33.0000	100.3910	110.3910
29	Hukuang (Hupei–Hunan)	湖廣全圖	Hou Quang	24.5000	33.5000	107.8910	116.3910

Note: Table 1 summarizes the map sheets in the order described here and includes the map number, name on map (as written by Walter Fuchs), Chinese map name (traditional characters as on the maps), name used by du Halde (du Halde 1735), and bounding box in four columns, as estimated by the present writer.

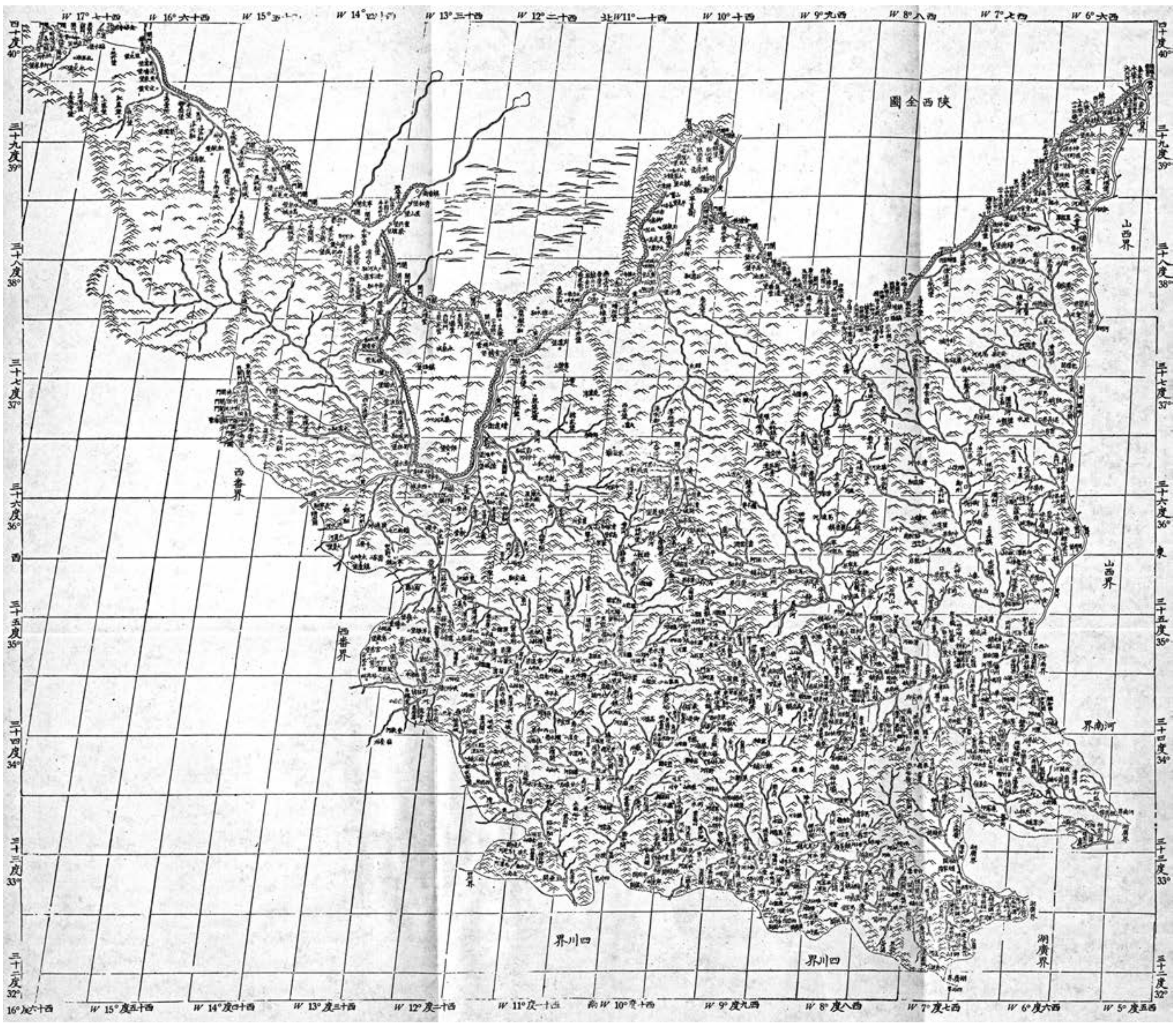

Figure 1. Scanned image of ShanGan Province showing Sinusoidal grid and annotations of Latitude and Longitude from the Beijing Meridian
Source: Regis and others (1941), Reproduced from the Geography and Map Division of the Library of Congress.

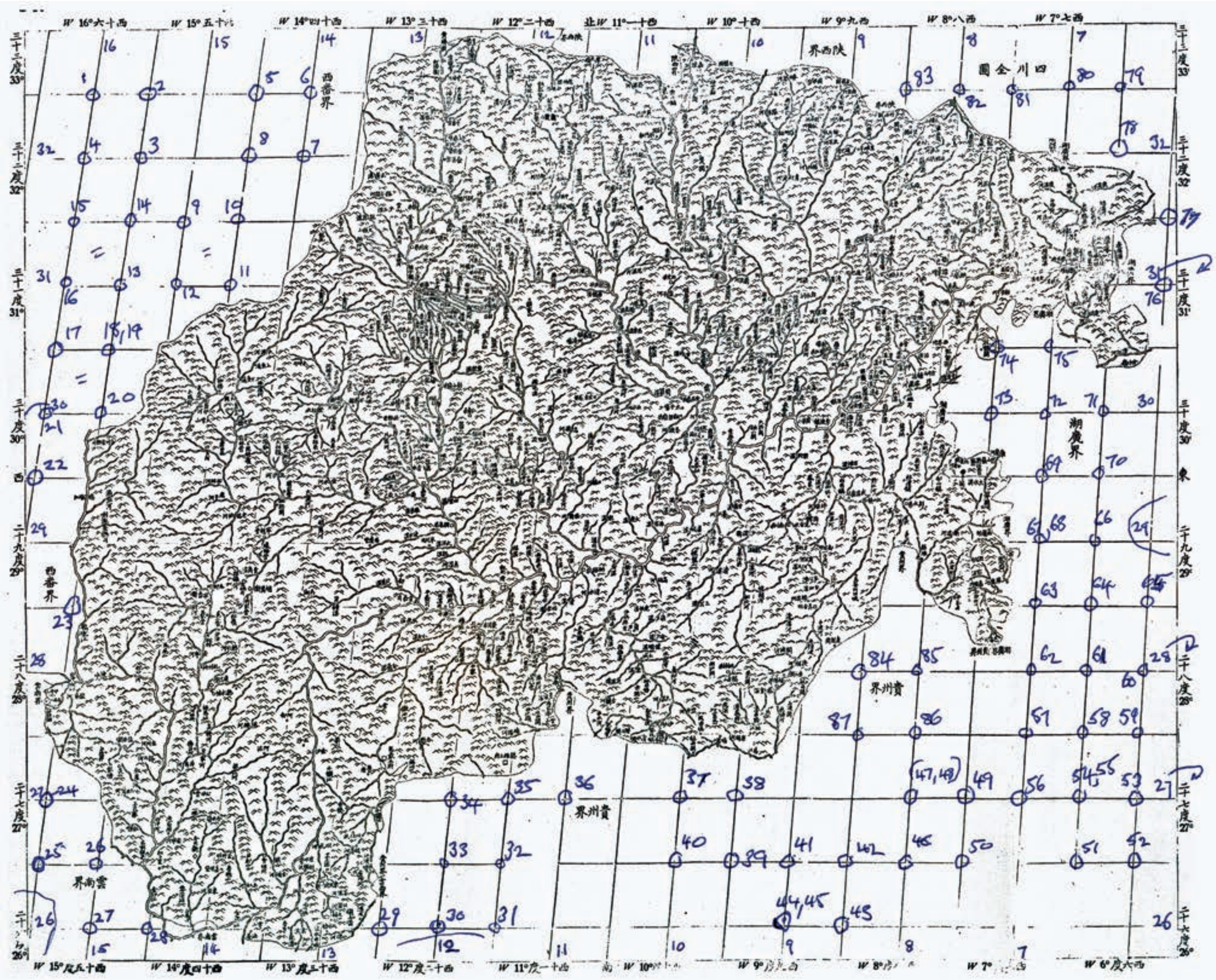

Figure 2. Scanned image of Sichuan Province with records of the crossing points selected to model the projection
Source: Regis and others (1941). Reproduced from the Geography and Map Division of the Library of Congress. Overlay markings by author.

Using measurements of page and neat line sizes made by the US Library of Congress (Ed Redmond, Geography and Map Reference Specialist, Library of Congress; personal communication) it was confirmed that the scans were close to precisely 400 dpi in both X (across the image) and Y (down the image) directions. This means that the spacing of the dots (which become image pixels) on the page was 0.00635 cm and the dots had equal size in X and Y. This can be taken as the size of a pixel on the original page and will be used to determine the scale of the maps.

SAMPLING THE PROJECTION GRID

To estimate map parameters, first, a relatively large number of the crossing points for the grid were located in the image and recorded in terms of the sample number (number of dots from the left-hand margin) and line number (number of dots down from the top margin line). Samples were also collected using significant features on the common borders with the other map sheets. Each crossing point corresponds to a latitude and longitude as read off the sides of the map.

Figure 2 shows the scanned map for the province of Sichuan. The edges have been trimmed so that only the map and the coordinates around the edge are visible. The geographic coordinates are indicated by the Chinese characters, with English added to the left on top and bottom and underneath on the two sides. On the top line of Figure 2, 九西 or W 9° indicates the longitude is 9° west of the Beijing meridian. On the sides of the map, the latitudes vary from 26 to 33° north of the equator. As with other maps, the folds have produced distortions locally and the original block printing is also of variable quality – but names are mostly clear after a zoom to full resolution. Figure 2 also shows the set of crossing points selected and marked on the copy of the map as cross reference. The grid is not visible within most of the mapped area, so there are none used inside the province. To the west of Sichuan, the annotation reads "西番界," or border with the western (Tibetan ethnic) tribal areas.

CALIBRATING THE PROJECTION

Let latitude be represented by ϕ and longitude by λ. A projection is a mapping with specific properties (especially 1–1, invertible and normally also differentiable) onto a Euclidean plane with coordinates (x, y) such that (Pearson 1990)

$$x = f_x(\phi, \lambda)$$

$$y = f_y(\phi, \lambda).$$

This allows given latitude and longitude values on maps to be converted into (x, y) coordinates on the projection plane, and the inverse model allows points of the projection plane to be converted into equivalent values of latitude and longitude. The general theory of map projections, including what they preserve and what they distort, is well covered in the book by Frederick Pearson II (Pearson 1990). After many years when the specific projection used by the Jesuit Brothers seemed unclear, the identification of the projection with a sinusoidal projection (Pearson 1990) was demonstrated by Wang Qianjin (Wang 1991). He showed that the projection of the maps in the copperplate edition can only be sinusoidal. He uses two primary pieces of information, convergence of the meridians and the fact that parallels of latitude are horizontal parallel lines and equally spaced. Equivalent to the convergence of the meridians would be to note that the steps between intersections of longitude on any one parallel of latitude are equal with value of cos ϕ times the distance between the adjacent parallel latitudes. Wang established their relationships by direct measurement. He also used his measurements to show that the scale of the copperplate maps (which had 1° spacing in latitude and longitude) is about 1:1.4 million. Recently, Lu and others (2011) repeated Wang's study using statistical hypothesis testing and proposed such methods as general tools to confirm the identity of projections in ancient maps.

The invertible equations for the sinusoidal projection are (Pearson 1990)

$$x = sR(\lambda - \lambda_0)\cos\phi$$

$$y = sR\phi.$$

In these expressions, apart from what has already been defined, λ_0 is the reference longitude, R is the radius of a spherical earth, and s is the scale factor. In the present case, λ_0 is the longitude of Beijing. The sinusoidal projection is based on a sphere. It is well known that that the form of the earth is better represented by a spheroid, and among common modern spheroids is the WGS84. It is consistent with GPS and most maps now take it as the base form. However, at the time the map was developed, the earth was still regarded as a sphere. For the present work, the authalic sphere (the sphere with the same centre and the same surface area as a spheroid; see Pearson 1990) for the WGS84 spheroid with a radius of 6,371,007 m has been used. If the scale factor is 1.0, then the units of the coordinates are metres on the sphere, and if the coordinates are (e.g.) cm on the printed map page, "s" will represent the map scale that converts into that coordinate frame.

The zero of the (x, y) coordinate system for the projection is at the crossing of the reference longitude and the equator. As a whole earth projection, it is very convenient. It has the nice property of being an equal-area projection, but distance and angle distortions become large away from the middle of the map, with the map only being conformal at the reference longitude at the equator. The sinusoidal projection was, however, especially convenient and suitable for the mapping exercise carried out by the Jesuit Brothers. It matched their survey model, with 0.5° of latitude north or south being a constant distance on the surface and parallels having the distance corresponding to 0.5° of longitude east or west being cos ϕ times the distance between the parallels. It was also a convenient global projection for storing, transporting, and re-projecting the information later, although Edward Cave (Cave 1741) did not seem to appreciate this.

This leads to a simple method for calibrating the projection for a given map. Supposing that the scanned grid of samples and lines (j, i) is a scaled grid on the Euclidean (x, y) plane, it is possible to model the coordinates as

$$x = x_0 + h_x \times j$$

$$y = y_0 + h_y \times i.$$

For the set of image sample and line values (j, i) for grid crossings there is a corresponding set of latitudes and longitudes (ϕ, λ) read from the map. These latitudes and longitudes can be converted into sinusoidal coordinates (x, y) in metres relative to a central meridian at Beijing. The factors (x_0, h_x, y_0, h_y) that best calibrate the above equations to provide sinusoidal (x, y) can be computed through minimum RMS error. The four fitted parameters provide a model for the projection. In particular, the scale parameters (h_x, h_y) indicate the scale of the maps. Errors here will arise mainly from errors in drawing the grid on the paper, paper stretch, printing errors, and uncertainties due to paper folds.

The grid crossings tell us nothing about drawn map accuracy – just about the initial drawing and representation of the projection grid. With accurate mosaics in mind, the common borders between provinces were

Table 2. Summary of fitted projection coordinates and errors of model fit at grid

	x_0	y_0	hx (m)	hy (m)	x_err (m)	y_err (m)
Shangan	−1,544,752.00	4,461,094.77	123.08	−122.78	637.64	1115.93
Sichuan	−1,575,148.77	3,685,246.54	123.06	−123.20	2103.73	823.48
Huguang	−838,352.05	3,736,215.42	123.66	−123.02	811.16	1240.87
Henan	−609,718.94	4,143,776.82	122.43	−122.26	744.63	556.62
Shanxi	−602,587.95	4,568,284.87	122.90	−122.39	525.74	625.95

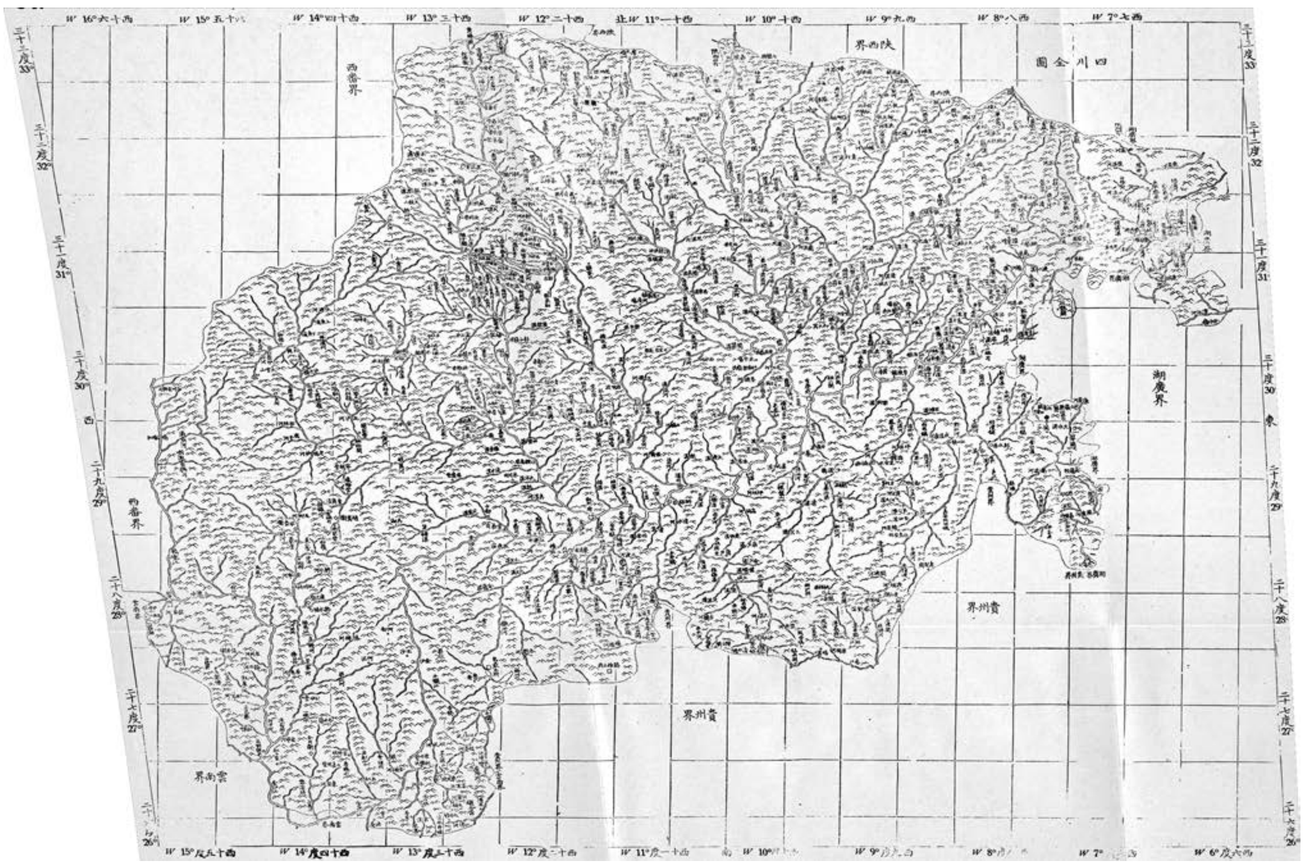

Figure 3. Sichuan province re-projected to geographic projection showing new grid lines as orthogonal equally spaced parallel lines
Source: Regis and others (1941). Reproduced from the Geography and Map Division of the Library of Congress.

sampled for specific features that were identified on adjacent maps. The constraint that these must be equal in the whole projection is quite a strong constraint and involves the drawn map data themselves as well as relationships between map sheets. These types of constraints were applied to the five maps used here. The fit is made in the projected space and so errors can be expressed in metres.

A summary of the results obtained in the five maps has been presented in Table 2. The units in Table 2 are metres, and most average map sheet RMS errors are less than or on the order of 1 km. Exceptions in Sichuan and Huguang were associated with fold distortions that were off the drawn map areas. These errors are much less than the 5–10 km (and greater) errors that will be seen later for true place errors. The fits between shared sections of province borders are not shown separately, but were again much less than the grid point errors, indicating very good prospects for successful mosaics.

The scale factor (s) is $s = \text{scan\_dot\_size}/\text{pixel\_size}$ and scale is normally written as $1{:}1/s$. Therefore, since the dot size established before was 0.00635 cm and the average pixel size above is 122.897 m, this gives the scale as about 1:2 million or more precisely 1:1,937,000. These parameters also allow the images to be geo-referenced for a sinusoidal

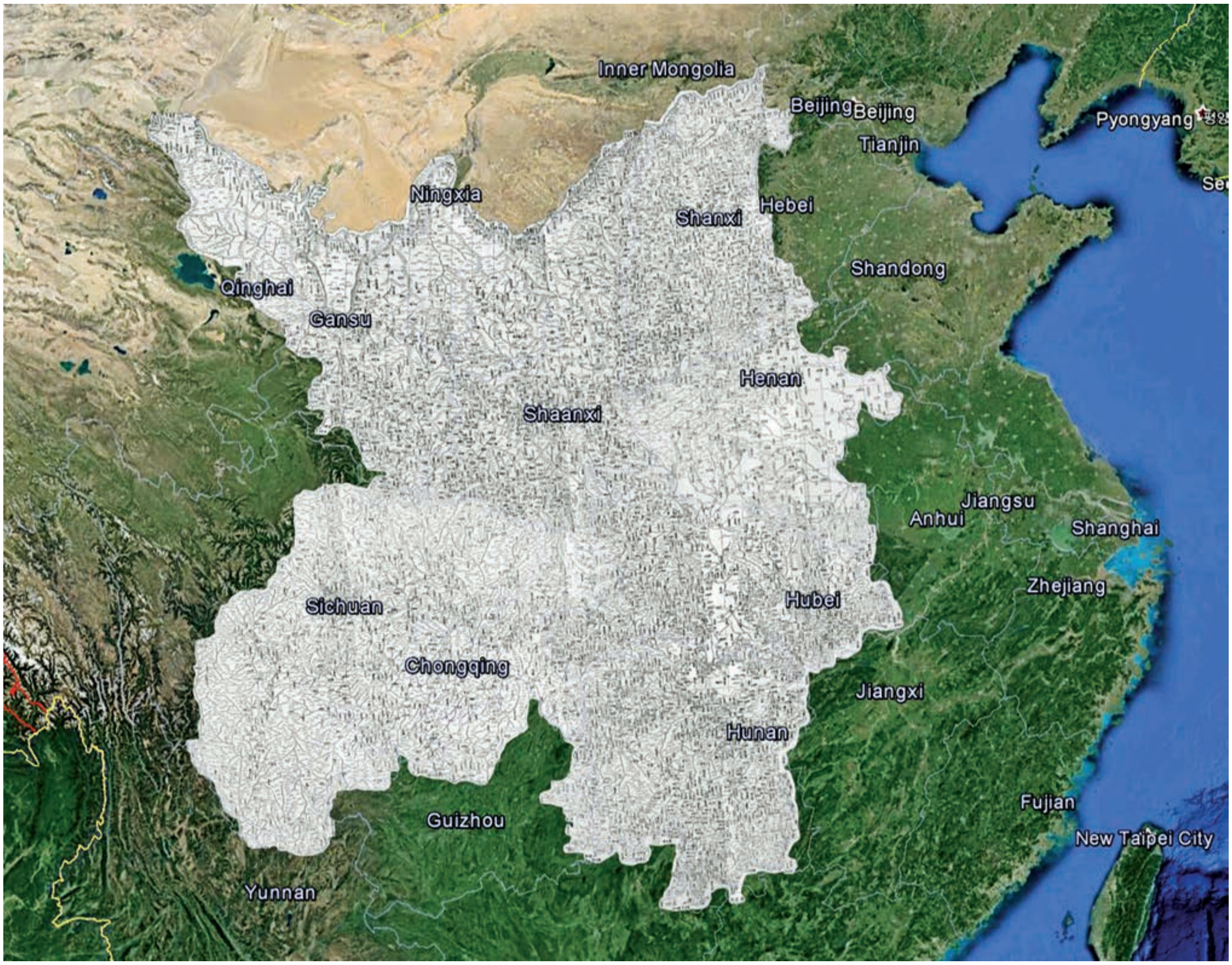

Figure 4. A mosaic of five provinces presented in geographic coordinates as a Google Earth super-overlay
Source: Map data © 2015 Google images by Landsat/Copernicus.

projection and (if desired) changed to another projection. For display in Google Earth it is wise to change the projection to geographic, as in Figure 3. The grid is now parallel, equally spaced, and normal in the vertical and horizontal directions. Grid cell sizes are the same in each direction. Google Earth and similar software can drape this kind of image properly over the Earth sphere.

FORMING MOSAIC IMAGES

A mosaic of all of five province maps was made into a single image. The boundaries were fully digitized as a polygon around each province in each map sheet and the mosaic was formed in the original Sinusoidal projection. The boundaries matched very well in all cases, possibly due to the added constraints between adjacent map sheets. Outside of the boundaries the data value was set to "null" (zero,) allowing the mosaic software to see through in this area to an image under a given province image. Finally, the mosaic was re-projected to geographical form and saved as a TIFF file. The super-overlay software used here (Klokan Technologies, https://www.klokantech.com/) allows the outer "null" areas to be transparent and therefore create a convenient image for presentation. The overlay can zoom up to seven levels, so that at the finest detail the characters that are printed clearly can all be read from this map. Figure 4 shows an example of the general view of the super-overlay in Google Earth.

Mapping Accuracy of the Five Maps

PREPARING THE CONTROL POINTS

In his book, du Halde (1735) provides a gazetteer with a separate table for each mapping region (e.g., provinces), listing altogether more than 300 places where the basic ground survey and astronomical measurements were combined to calculate latitude and longitude. The places in the gazetteer can also be used to assess the intrinsic accuracy of the mapping. Intrinsic accuracy is taken to be the size of differences between these places and known locations for the Qing period places corresponding to the ones in the gazetteer. As other places in the final map are most likely located by scaling information from existing local maps relative to the base plotting data, the direct total accuracy will be no better than this.

It was first important to assess if the points were accurately plotted on the maps. One possible way in which the maps were developed was to initially draw the grids, then to plot the control points according to the grids, and finally to scale in local data according to Chinese maps. When selected plotting positions of the base points were checked manually on the five maps used here, it was observed that they were normally very close. Wang Qianjin (Wang 1991) made a more detailed study of plotting positions for the copperplate map in the 15 "inner" provinces of China. He found that they were generally close but that there were occasionally quite large errors, which may be transcription errors in the positions in du Halde's gazetteer (du Halde 1735). He also found that errors in longitude were greater than errors in latitude, which could represent the greater difficulty of drawing the longitude grid accurately and of the manual interpolation, due to the convergence of parallels.

The reference data set used to estimate intrinsic accuracy was developed by the China Historical GIS (CHGIS) Project at the Fairbank Center for Chinese Studies at Harvard University in cooperation with Fudan University in China. It will be referred to in the following document as the "ChinaW" data set and comprises a collection of Qing period places at district level and above (dao, fu, zhou, xian, and ting) as they were in the period 1820–93. A few of the places in du Halde's gazetteer could not be found in the ChinaW set (as they were small forts or other places below ting level) and were not included. The longitude coordinates from Beijing were calculated assuming the longitude of Beijing was 116.391° east of Greenwich. There is an apparently surveyed line through the middle of the Emperor's palace and the Tian Tan Temple, but it is not quite a true-north meridian. The point chosen here is on this line halfway between the reference points.

In the du Halde gazetteer, the 300 or so points are places where the latitudes are based on astronomical measurements and where the longitudes are the result of geometric measurements – meaning survey and the method of triangles. The commentary by Fr. Jean-Baptiste Regis quoted by du Halde (du Halde 1735; Cave 1741) indicates that despite early enthusiasm for taking astronomical measurements, they later became concerned that astronomical observations for longitude would not be accurate enough due to the hard and somewhat rushed situation of their extended surveys. Fr. Regis wrote (Cave 1741),

> after mature deliberation we thought it best to [only] use the method of triangles, all others appearing to us to be not only too tedious, considering the vast extent of the countries of which the Emperor wanted the map, but scarcely practical on account of the towns being so near to each other; since it is certain that the least error, occasioned by the pendulum going wrong, or the immersion of one of Jupiter's satellites not being accurately observed, would cause a considerable error in the Longitude.

The method of triangles is best used between places close enough for the flat earth geometry to apply. As the route continues, increments in latitude and longitude are calculated and accumulated. As latitudes are measured at each major place, there are some feedback corrections that allow independent and relatively reliable latitudes to be provided at the base locations. However, unless at least one reference place is included where absolute longitude is known, the value will always only be relative to the starting point of the survey. Some fixed points of reference (especially for longitude) in other places are therefore essential to maintain the baseline of the survey and peg the widely separated maps into an accurate framework. Because of this, Fr. Regis continued,

> We therefore contented ourselves with observations of the Moon and satellites of Jupiter made before our time by members of our Society, though we rejected a few because they did not agree with our measures, on account of some small error as to time in the observation, which too often happens to the most experienced.

Because the du Halde gazetteer lists the primary mapping points for the maps and because these points can be identified with known places today, they together provide a means of assessing the intrinsic accuracy of the maps in the five provinces. Given the caveats expressed by Fr. Regis, it is expected that longitude errors in the astronomical observations will be the most likely source of high intrinsic errors.

OVERALL INTRINSIC ACCURACY

Using 116.391°east of Greenwich for the reference longitude of Beijing, the latitudes and longitudes for the maps and control points were available in equivalent coordinates to the ChinaW points and to modern maps and vice versa. Using the ChinaW set of dao, fu, xian, zhou, and ting designators as they existed in 1820–93, the place names were identified and the coordinates matched with those in the du Halde gazetteer. In this case, the statistics measure true accuracy on the ground.

Taken overall by province, the errors (in units of kilometres) are listed in Table 3. The errors in latitude and longitude have been converted into kilometres in the inverse way to how the Jesuit Brothers converted li to degrees. The average error and standard deviation of error

Table 3. Summary of errors (km) for five provinces based on ChinaW database

Short Name	Chinese Name	Du Halde Name	Av Lat Err	SD Lat Err	RMS	Av Lon Err	SD Lon Err	RMS
ShanGan	陝西	Chen Si	−0.800	9.024	9.059	1.660	21.325	**21.389**[a]
Shanxi	山西	Chan Si	−**3.530**	8.465	9.172	**9.318**	7.689	12.081
Henan	河南	Ho Nan	−**7.824**	3.778	8.688	−1.561	10.228	10.346
Sichuan	四川	Se Tchuen	0.310	9.541	9.546	−0.047	5.033	5.033
Huguang	湖廣	Hou Quang	1.012	7.556	7.624	−**21.090**	10.467	**23.545**[a]
Mean all			−2.166	7.673	7.973	6.092	10.948	12.529

Note: Bold numbers indicate errors to be investigated.
[a] These two RMS values point to ShanGan and Huguang as the most significant locations of errors.

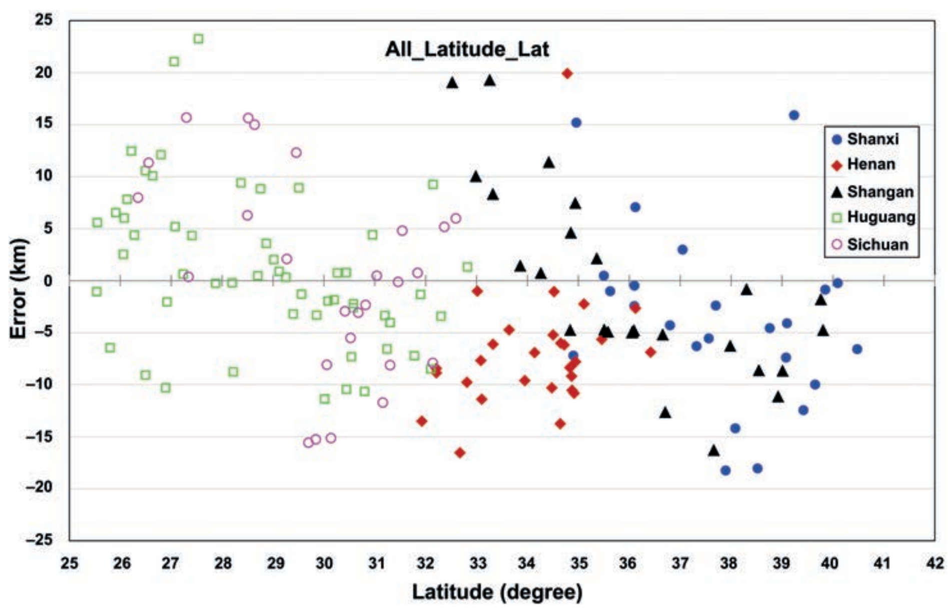

Figure 5. Errors for five provinces in latitude as km on the ground plotted against increasing latitude (from south to north)

are both provided to identify possible regional bias error and RMS is the total error (RMS = sqrt(mean^2 + StDev^2)).

These overall figures do not identify the locations of specific areas with significant errors, but the basic message is clear. The overall RMS seems to be about 8 km for latitude and 12 km for longitude. ShanGan and Huguang seem to have significant longitude errors, and if they are left out, the overall average RMS for longitude drops to about 10 km. Shanxi seems to have some bias error in both latitude and longitude, and Henan some latitude bias error. Huguang has a very large longitude bias error as well as RMS. Sichuan has very low RMS error in longitude, but may have issues with some of the measured latitudes.

From these samples, the intrinsic accuracy (RMS) of latitude seems be about 8 km, which is about 0.05° or 5 minutes. This includes factors not related to the original measurements, and so perhaps the RMS accuracy of only the Pole Star measurements could be about 0.025°, which would mean they were done competently in a very difficult survey environment. To provide some reference, in 1714 Newton was attempting to devise how to use the moon to determine longitude (Dunn and Higgit 2014). His objective was to achieve measurement to 2 minutes of arc (about 0.03°), but he could not do it even with the facilities he had available in Europe. This represents an error on the ground of about 3.3 km. The Jesuit Brothers would have had tables for the declination of the Pole Star, which were essential at the time, as the Pole Star was further from the true north point then than it is today. But how good the tables of declination were is not known. Because of all the uncertainties, it seems that measured latitude is no better than may be expected but is still very variable. It seems similarly variable in all areas sampled here, as indicated in Figure 5.

Figure 5 plots only latitude error in kilometres by latitude. The scatter of error in Figure 5 is high at all latitudes. There seems to be a trend of error with latitude, but in

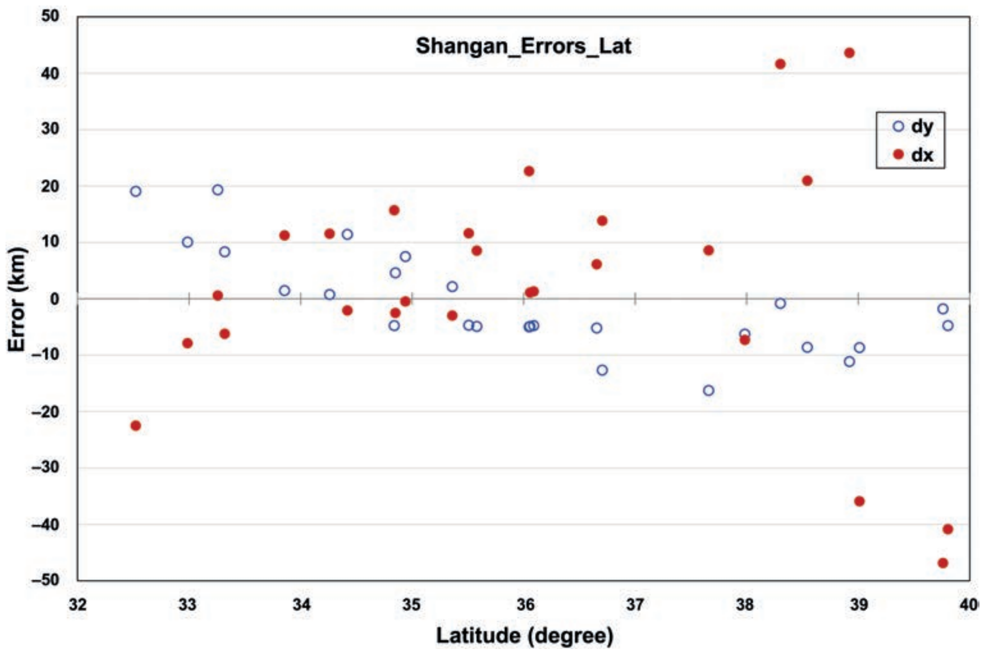

Figure 6. Errors (km) in latitude (*dy*) and longitude (*dx*) for ShanGan plotted by latitude

the scatter of data it would be hard to say it is significant. It may have something to do with the Pole Star estimate, but more information would be needed about the methods used to decide this. Despite the scatter, the present writer's opinion is that the latitude errors (even the −20 to +25 km min to max range) would not have been unusual in 1718, even in Europe. Consequently, apart from areas where surveys through mountainous terrain seem to have led to bias in both latitude and longitude, the main source of significant regional error seems to have been the difficulty of measuring accurate longitude differences to Beijing and the propagation of distortion this creates. The provinces where the RMS error is most significant (mainly in longitude) are the two combined provinces of ShanGan (Shaanxi and Gansu) and Huguang (Hubei and Hunan). It turns out that these errors are specific and can be regionally located.

LOCAL ERRORS IN SHANGAN

If the errors in both latitude and longitude (in units of km) are plotted only for ShanGan province, the result is as shown in Figure 6. Note that in graphs where both latitude and longitude, or simply longitude, are plotted the scale used is from −50 to 50 km to accommodate the greater error found. Previously, when only latitudes were plotted, a scale of −25 to 25 km was quite sufficient.

In Figure 6, the blue dots are latitude errors and the orange dots are longitude errors. They are plotted against latitude. Again, there is an interesting, apparently systematic trend in latitude. Although most longitudes plot in the 25 km range, there seem to be two areas with very large longitude errors. One is in the north (highest latitudes) and one in the south. In the south, the same places also seem to have larger latitude errors. If the errors are located on the map it is found that the source of the northern problems is along the Great Wall and the southern problems occur in the Han River Valley.

The errors in latitude and longitude for places in ShanGan near the great wall (a limited latitude range) were extracted and plotted against longitude in Figure 7. The places involved, from west (left hand) to east (right hand), are Jiayu Guan (嘉峪関), Su Zhou (肅州), Gan Zhou (甘州), Xining Zhou (西寧州), Liang Zhou (涼州), Lan Zhou (蘭州), Zhong Wei (中衛), Ningxia Wei (寧夏衛), Yulin Wei (榆林衛), and Shenmu Xian (神木縣). The linear change in longitude error (orange dots, *dx*) is quite striking. However, south of Lanzhou (where the longitude error is smallest), this behaviour is not found. The latitude errors (blue dots, *dy*) are similar in magnitude to those in most other places and show no systematic bias.

As to what is occurring, it is known from the book by Étienne Souciet (Souciet 1729) that Jiayu Guan and Su Zhou at the western end of the Great Wall were places where astronomical measurements were most likely originally made for longitude, and it seems there is significant error in both places. It is possible that many places along the Great Wall were not revisited during the provincial surveys, to save time, as the Great Wall had already been mapped from Jiayu Guan in the west to Shanhai Guan in the east (at the sea), so the errors may have been made at that time. Lanzhou seems to have been visited during the new survey, and possibly there was a good astronomical

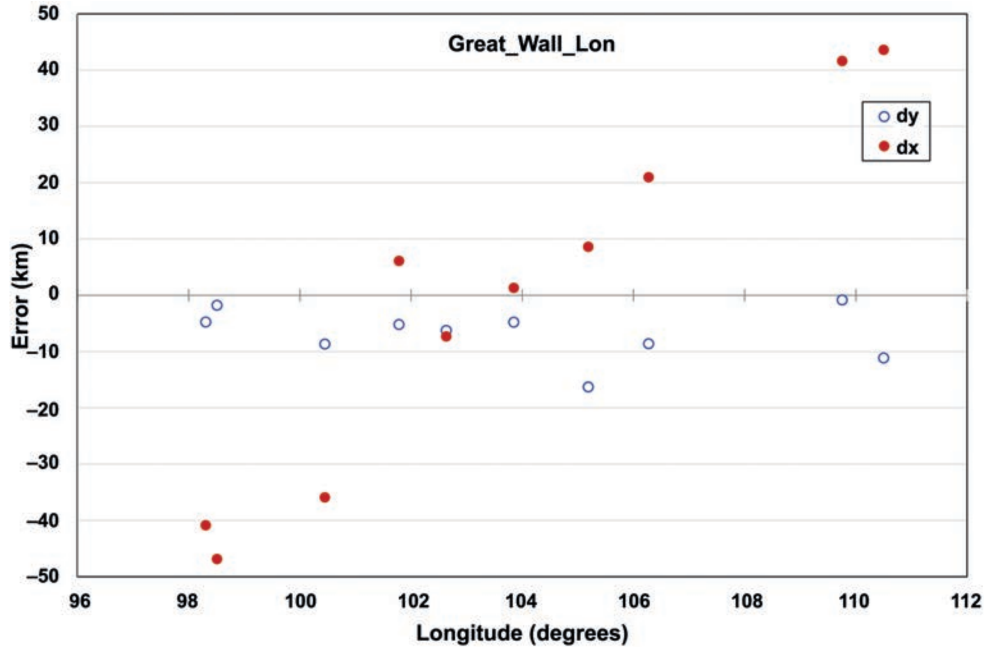

Figure 7. Errors (km) in latitude (*dy*) and longitude (*dx*) for ShanGan places along the Great Wall plotted by longitude (east of Greenwich)

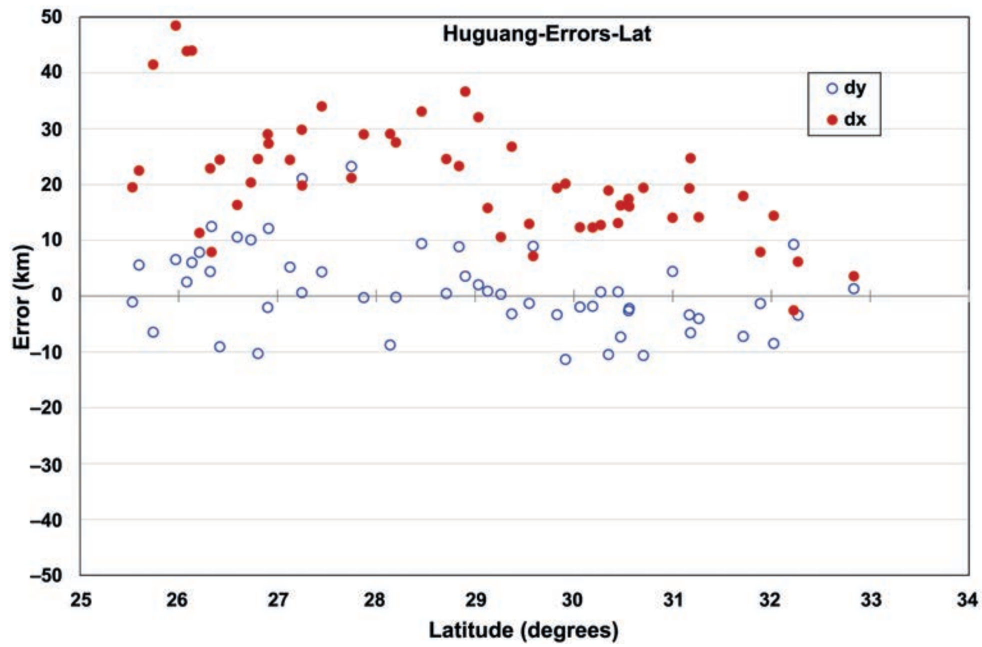

Figure 8. Errors (km) in latitude (*dy*) and longitude (*dx*) for HuGuang province plotted by latitude (small in south and large in north)

measurement there. It is curious that these very large errors do not continue to the places near the Great Wall from ShanGan into Shanxi. However, the situation shows clearly how errors in anchor points can easily propagate into a surrounding region and how a better anchor point can quickly control the map.

The remaining larger errors are in the Han Valley and mainly at the eastern end near Xing'an Fu, across the Qinling mountains. These led to distortions in the Han River (a shift of about 20 km) that were exacerbated during map drawing by errors in the neighbouring province of Huguang. Apart from the Great Wall section and the

Han Valley, the map accuracy is excellent – most likely due to good longitude observations at Xi'an and perhaps Lanzhou. The linear change in longitude across the Great Wall is possibly consistent with the measure of the new standard li being in error in this early section, but establishing this will require further studies. Since the east end of the Great Wall at Shanhai Guan was located astronomically with great precision, it seems from the present work that the very large errors are confined to ShanGan.

LOCAL ERRORS IN HUGUANG

If the errors in both latitude and longitude are plotted in kilometres for Huguang by latitude, the result is as shown in Figure 8.

Again, in Figure 8, the latitude errors (blue dots, *dy* in km) show some possible trend with latitude, and the general latitude errors are no greater in magnitude than in other places. However, there is a dramatic difference in the longitude. For latitudes lower than 30°, the errors are very large, reaching to over 40 km. It seems as though there is a large bias in the error, but removing a constant bias (e.g., changing the location of Beijing) makes errors at latitudes higher than 30 degrees too large. The problem is not simply fixed by change in baseline. Huguang was part of a survey that started with Brothers Fridelli and Bonjour going to Sichuan and then Yunnan. Owning to the death of Fr. Bonjour and the illness of Fr. Fridelli, Fr. Regis went to Yunnan and, with a recovered Fr. Fridelli, completed Yunnan and moved on through Guizhou and Huguang. A more complete study must therefore include all of these provinces and consider the survey sequence and route.

The errors discussed here are the main sources of the distortions that occur in the five-province mosaic. The most obvious distortion is in Huguang, where the southeast corner of the mosaic is shifted significantly, as well as locally in the Yangtze Valley, the Great Wall, and the (Upper) Han River areas. However, in most of the mosaic, with average errors of 8 km in latitude and 12 km in longitude, it still provides an amazing product for 1721. The detail of places on the 1721 maps and their historical snapshot of China's administration make the value very high. It is planned to extend the work to the 15 "inner" provinces and include the base points in the model development to identify transcription errors. Then problem anchor points can be identified by analyzing the provinces in groups as they were surveyed. If warping the mosaic to fit a modern map is of particular value, it can be done based on a mosaic in the original projection space. It would be possible to warp the map as described by Akin and Mumford (2012) or use Georeferencer or similar software (Klokan Technologies, http://www.georeferencer.com/).

Discussion

THE PROBLEM OF LONGITUDE

The main limitations on the accuracy of the Kangxi maps were from the astronomical measurements of longitude. Fr. Regis was right to say that for places close together, the survey was more accurate than using the astronomical measurements. Provided the incremental distances between base mapping places for which the survey was done were not more than 2 or 3° of latitude or longitude, the geometry could be treated as Euclidean and the estimated differences in latitude and longitude would be accurate – unless the terrain was very rough. But the error would build up continually if this were done to far places, and if there was a new starting point, the connection would be lost entirely. The whole map therefore has to be fixed by a good set of anchor points where the latitude and longitude are accurately known. The problem was that in the case of longitude, they were not always available for the Kangxi map, and some that were available seem to have had significant error.

However, this was not due to poor workmanship by the Jesuit Brothers. Since ancient times, longitude had been very hard to estimate by any practical method. The problem was most critical at sea, where currents and winds were difficult to guess and errors of hundreds of kilometres were quite easy to make. Errors of more than 100 km also occur in the China maps of Martino Martini (1655), produced only about 50 years earlier. In the period 1707–19, the latest European methods were astronomical and were brought to China by the Jesuit Brothers, who applied them as accurately as possible in the harder survey conditions. But 50–100 years later, longitude on the sea and the land was being determined accurately with the help of robust clocks – or chronometers (Dunn and Higgit 2014). The reason is that the best measure of longitude is time. Local time (or local apparent time, LAT) is based on midday being when the sun is at its highest in the sky. A pendulum clock can be set at that time to record the LAT at other times. The difference in LAT between two places at the same instant is precisely related to longitude difference. But it was hard to move clocks and have them still keep accurate local time at the reference location – especially at sea. An alternative was for people to observe the same astronomical event at different places and record the times at two places for the same event in LAT. Later they could be compared. But as Fr. Regis noted,

> The Observations of the Satellites require, not only more Time and Accuracy, but also Telescopes of the same Size, and if I may so speak, the same Eyes in the Observer and his Correspondent; for, if one sees them ever so little sooner than the other, some Error will inevitably happen.

An alternative, which was becoming standard, was for regular astronomical events to be predicted and the times of observation at (say) Paris in the future calculated into tables. Then only a single observer was needed. But at that time, tables were not always accurate and LAT was not easy to measure. The recorded data listed by Souciet (1729) show wide variations between observations and alternative tables. Even if the measurement were made carefully, the longitude difference from Paris would not have been very accurate when shifted to a longitude difference from Beijing. In Europe at the time there were many observatories that careful observers maintained for many years, but even so, it was not until modern chronometers were developed that accurate and easily obtained worldwide longitudes became a reality (Dunn and Higgit 2014). Based on the story of the equivalent survey of France reported by Konvitz (1987), it seems that except for a certain added roughness due to the difficult conditions faced by the Brothers and the presence of some poor astronomical estimates of longitude in specific areas, the accuracies found in these maps are simply what can be expected for the time.

TRIANGLES AND TRIANGULATION

The method of ground survey carried out by the Jesuit Brothers has sometimes been referred to as "triangulation." However, the present author feels it is not certain enough to call it such, but rather, it is better to simply call it the "method of triangles" or adjusted transects. As described in Konvitz (1987), the French national mapping survey at the same time was certainly a very detailed application of triangulation as it had developed in recent years, and because of this, it took about 70 years to complete, compared with the Jesuit Brothers taking 10 years to cover a much larger and more difficult area. The 1965 *Admiralty Manual of Hydrographic Surveying* (Hydrographer of the Navy 1965) provides a detailed and practical discussion of triangulation (primarily measuring angles) and trilateration (measuring the sides of a triangle) combined with data adjustments as a survey technique. The properties of 2D and 3D triangles allow already relatively accurate surveys to be improved by mathematically adjusting the interlaced mesh of data. This method also allows bad data in surveys to be flagged as needing a repeat of measurements. The Jesuit surveys sometimes incorporated aspects of triangulation in measurements of points to the side and returns to the same place. Du Halde (du Halde 1735; Cave 1741) includes notes provided by the Jesuit Brother Fr. Regis, who wrote,

> Another Method, which we judged ought to be employed for greater Precision, was to return to the same Point, already determined, by different Ways, from a considerable Distance, working according to Rules. For if by the last Essay you find the same Situation, the Exactness of the preceding Operations will be proved in some measure to a Demonstration. When in measuring we could not return to the same Point, our Method was, as we passed near the great Towns already marked down, or other fit Places, to look out for remarkable Towers, or Mountains that commanded them; and from time to time we measured, to see if the Distance resulting from the Operations (when corrected) agreed with the actual Measure.

However, the majority of survey lines mapped in practice seem, in fact, to have been what surveyors would call adjusted transects (Hydrographer of the Navy 1965). That is, they involved segmented survey lines adjusted for height and bearing variations between major places where astronomical data were taken. The triangles used seem to have been the incremental length changes in parallel and meridian either for a segment or for the whole survey. The result of this approach would not have been as accurate as the triangulation carried out in France, but it was most likely the only way it could have been carried out in the time. At an end place, they may have also computed the bearing of a primary point to the last and/or added the incremental changes between sub-transects. The distances could be resolved (by Pythagoras' Theorem) into incremental distances north or south and east or west between the end places by reference to a compass. With a sufficient number of accurate astronomical measurements and adjustments, the result would be quite accurate. An added advantage was that the elements of this task and effective methods needed to carry it out would have already been known to the Chinese surveyors who did much of the work. The principles of measuring distance "as the bird flies" had been outlined in ancient times by Pei Xu (裴秀) during the Jin Dynasty (晋朝, 265–420 CE). Pei Xiu formulated six basic principles of such surveys.[1] According to the text of Cao (1992), Pei Xiu wrote as follows (see the Appendix for the Chinese text):

> *Map making has six basic principles:*
>
> (1) The graduated scale (Fenlü), by which the extent of the map is measured;
> (2) The standard grid (Zhunwang, of equally spaced parallel lines in two dimensions), which ensures correct relationships between the various parts of the map;
> (3) Measuring distances from others (Daoli), which ensures an accurate value (triangulation);
> (4) Accounting for terrain relief (Gaoxia);
> (5) Accounting for slope and aspect (Fangxie);
> (6) Accounting for winding and straight (Yuzhi).

The last three of these principles account for terrain variations so that distortions are removed.

> If you have a map but no graduated scale, you cannot establish what is near and what is far. If you have a graduated scale but no standard grid, what you can represent in one place is

lost in another. If you have a standard grid but not correct relative distances, then if you try to combine different regions they will not be consistent. If you have correctly proportioned distances, but do not apply the last three principles, then many pathways will not be consistent in relative distances so we can say all six principles must be taken into account.

In modern terms, the survey is adjusted "to the geoid," which is assumed to be a locally planar area. Of the six, the Daoli is especially important, being the basis of triangulation, and Pei Xiu says that, without a combination of this and the final three corrections, accurate relative distances cannot be determined. It seems that these principles were all usefully and regularly applied in the Kangxi map surveys – although there is not a lot of evidence that they had been so regularly applied in traditional Chinese maps since the time of Pei Xu.

Conclusions

The Kangxi maps were an outstanding application of emerging European land survey and mapping techniques combined with traditional Chinese surveying to map a large area of East Asia. They also provide us today with significant information about the political geography at the end of the reign of the Kangxi Emperor. At the time, the most significant outcome was the incorporation of the information into western atlases at accuracies commensurate with those of European land maps of the time. The set of maps, originally wood block printed in 1721 and later reprinted in facsimile by Walter Fuchs in 1941 (Regis and others 1941), has been preserved, scanned at high resolution, and made accessible to the public by the US Library of Congress (Geography and Map Division of the Library of Congress). The maps have been used here to study the projection parameters, scale, and map accuracy for the five central and western provinces of Shanxi, ShanGan, Sichuan, Henan and Huguang.

The projection parameters of the individual province maps, at near 1:1.94 million scale, were estimated and used to re-project and mosaic the maps to create presentations suitable (e.g.) to use in Google Earth, such as the mosaic illustrated in Figure 4 (Jupp n.d.). The accuracy of the maps was established using the gazetteer of mapping places provided by du Halde (du Halde 1735). In general, the error in the best mapped areas was about 6–8 km in distance equivalents for latitude and longitude although in most places, it was about 8 km for latitude and 10 km for longitude in RMS error. The large errors, which tended to be extended into adjacent regions by the method used, were in longitude, with some large regional distortions on the order of 20 km occurring in specific places and especially in the southeastern region of the set. The longitude errors seem to have been the result of inaccurate astronomical measurements and/or the sparse network of map control available to the Jesuit Brothers. But at the same time, in the development of such surveys, the situation would have been little better for European maps (Konvitz 1987). Longitude was to remain a significant problem in any survey for another 50 years (Dunn and Higgit 2014).

Similar exercises can be extended to other provinces or groups of provinces in the Kangxi map series for a more complete analysis of the maps. There seems to be significant bias that spreads through the southeast of China. However, to establish the full nature and source of this distortion, which seems to involve at least one bad astronomical observation of longitude, a study of the provinces surveyed together as specific groups by Jesuit Brothers could provide a sensible stratification.

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The scanned maps used in this article were re-printings of original maps by Walter Fuchs (Regis and others 1941). They were downloaded from the Library of Congress Geography and Map Division, Washington, DC 20540–4650, USA; Ed Redmond, Geography and Map Reference Specialist, US Library of Congress, measured the neat lines of selected maps to confirm the scale. Figure 4 is a screen image from Google Earth Pro software; Google and the Google logo are registered trademarks of Google Inc.; the KML super-overlay was produced by Klokan Technologies MapTiler Software. The author wishes to thank two anonymous reviewers for their support for the work and the *Cartographica* editorial staff for their improvements to the text.

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Note

1. A translation contributed by the writer of chapter 22 ("Geography and Cartography") of volume III of *Science and Civilization in China* (Needham and Wang 1971) provides a useful reference that aims to uncover the details of Pei Xiu's thinking. However, the present writer felt that for the discussion here it would be best to stay as close as possible to what Pei Xiu wrote, with minimal interpolation. Consequently, a revised translation using the source text as found in Cao (1992) has been made, which agrees with the above translator's identifications but aims for minimal departure from the original, using source material from original in the Jin Records.

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Appendix: Information Attributed to Pei Xiu

制图之体有六焉。一曰分率, 所以辨广轮之度也。二曰准望, 所以正彼此之体也。三曰道里, 所以定所由之数也。四曰高下, 五曰方邪, 六曰迂直, 此三者各因地而制宜, 所以校夷险之异也。有图像而无分率, 则无以审远近之差; 有分率而无准望, 虽得之于一隅, 必失之于他方; 有准望而无道里, 则施于山海绝隔之地, 不能以相通; 有道里而无高下、方邪、迂直之校, 则径路之数必与远近之实相违, 失准望之正矣, 故以此六者参而考之。然远近之实定于分率, 彼此之实定于准望, 径路之实定于道里, 度数之实定于高下、方邪、迂直之算。故虽有峻山钜海之隔, 绝域殊方之迥, 登降诡曲之因, 皆可得举而定者。准望之法既正, 则曲直远近无所隐其形也。

晋书裴秀传 (*Jin shu*. "Pei Xiu zhuan," quoted in Cao 1992)